M 40

BEACONSFIELD BY-PASS

WINDSOR END FOOTBRIDGE

CALCULATIONS

E.H. Frankland, M.I.C.E.
County Surveyor, Bucks. C.C.
County Offices,
AYLESBURY, Bucks.

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BRIDGE GEOMETRY

This section deals with the calculation of the required bridge spans from the equations of Motorway centre-line and Marginal strip; and of vertical alignment from the relevant Motorway and Embankment levels. All are related to the M.O.T. Design memoranda for bridge clearances.
A type of arched bridge is postulated, calculations made of arch geometry, and Computer data and print-out sheets presented for cross-section properties at 2 ft intervals.

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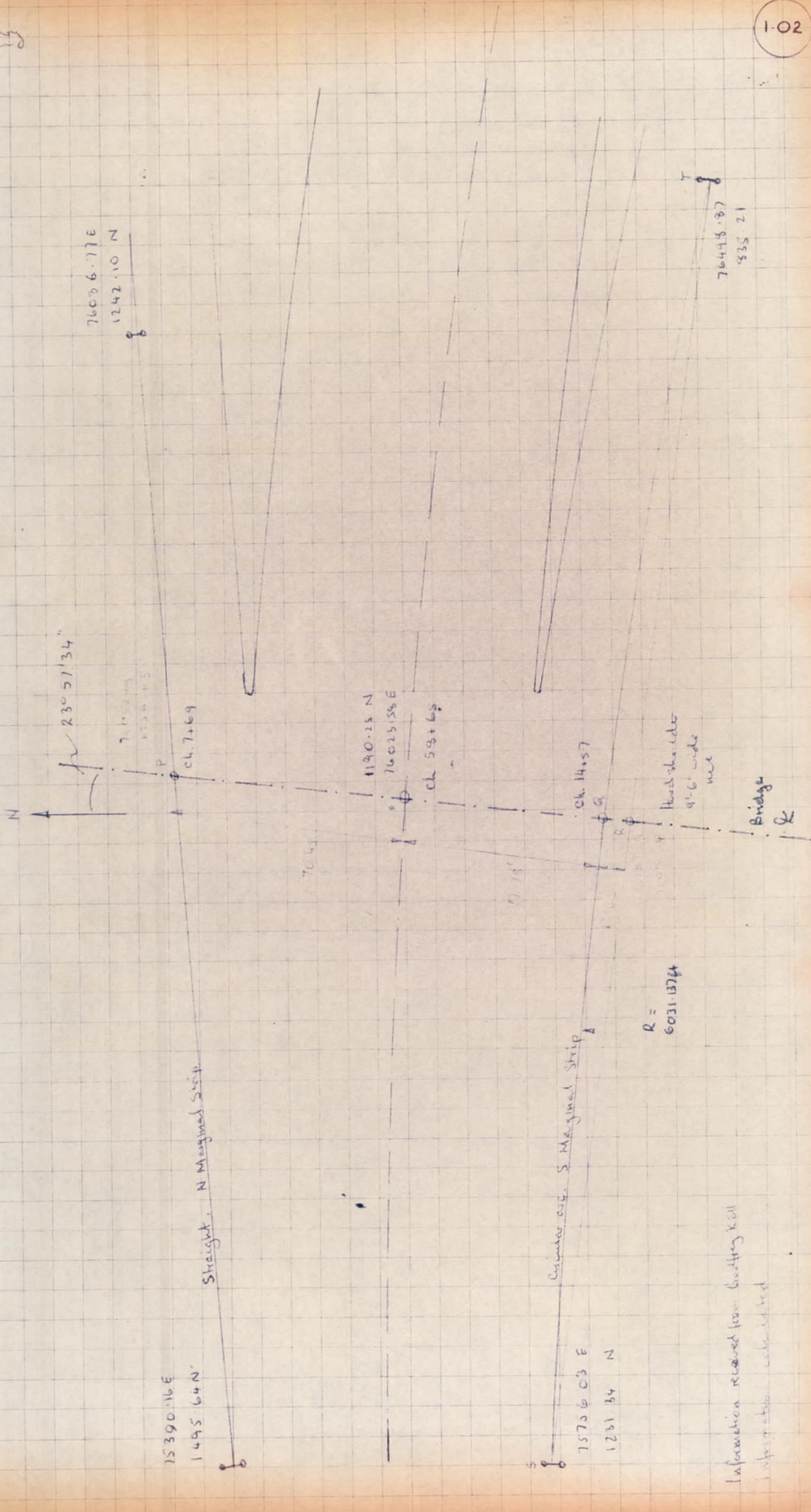
Summary



76036.77 E
1242.10 N
76448.87
835.21
N
23° 51' 34"
ch. 7+69
P
1190.25 N
76023.58 E
ch. 58+65
ch. 14+57
Q
R
Hard shoulder
9'-6" wide
Bridge
R = 6031.1374
Straight. N Marginal Strip
75390.16 E
1495.64 N
Curve etc. S Marginal Strip
75736.03 E
1231.34 N
S
T
Information received from Geoffrey Kell

Summary

Bridge ℄ $y = m_1 x + c_1$ $m_1 = 2.250323$ — ①
$c_1 = -169887.3606$

North Margin $y = m_2 x + c_2$ $m_2 = -0.3639626$ — ②
$c_2 = 28934.8386$

South Chord $y = m_3 x + c_3$ $m_3 = -0.533264$ — ③
$c_3 = 41629.3036$

Intersection of ① and ② (Point P)

① - ② $0 = (m_1 - m_2)x + c_1 - c_2$

$$x = \frac{c_2 - c_1}{m_1 - m_2} = \frac{28934.8386 + 169887.3606}{2.250323 + 0.3639626} = 76052.19$$

$$y = 76052.19 \times 2.250323 - 169887.3606 = 1254.6318$$

Length $OP^2 = (76023.38 - 76052.19)^2 + (1254.6318 - 1190.25)^2$

$= 4963.54827$

$OP = 70.45$ ft

Intersection of ① and ③ (point R)

$$x = \frac{c_3 - c_1}{m_1 - m_3} = \frac{41629.3036 + 169887.3606}{2.250323 + 0.533264} = 75984.93$$

$$y = 75984.93 \times 2.250323 + 169887.3606 = 1103.275$$

Length OR

$OR^2 = (1190.25 - 1103.28)^2 + (75984.93 - 76023.58)^2$

$= 9057.6034$

$OR = \cancel{95.17}$

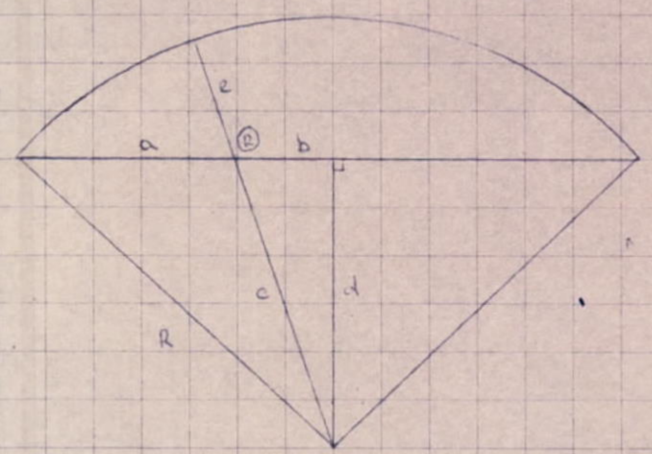
Length ST

$ST^2 = (75756.03 - 76498.87)^2 + (1231.34 - 835.21)^2$

$= 708752.5277$

$ST = 841.8785$

To Find e



$2(a+b) = 841.874$
$a+b = 420.987$
$R = 6031.137$
$a = 262.29$
$b = \cancel{158.70}$
$d = 6016.43$

$a^2 = SR^2 = (75756.03 - 75984.93)^2 + (1231.34 - 1103.275)^2$

$= 68795.5542$

$a = SR = \cancel{261.29}$

$d^2 = -(420.98^2) + 6031.14^2 =$

$d = 6016.43$

$c^2 = d^2 + b^2 =$

$c = 6018.5853$

Then $e = R - c = \cancel{12.59}$ ft.

And $OQ = OR - e = \cancel{95.17 - 12.59} = 82.58$ ft.

check that R (75987.08(6), 1108.13(6)) lies on OR and ST

Eq'n of ST $y = -0.533264x + 41629.3036$

sub for x $y = 1108.12$ - o.k.

Eq'n OR $y = 2.250323x - 169887.3606$

Sub for x $y = 1108.13$ - o.k.

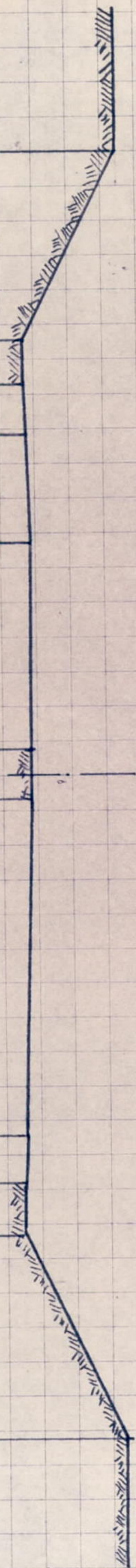
Calculations for Levels

(a) S.W. Slip Road. N. marginal Strip, = 320.12
@ ch. 14+57 S. marginal Strip = 319.47

(b) N.W. Slip road N. marginal strip = 320.98
@ ch 7+69 S " " = 321.64

Final Cross Section with Levels received from Godfrey Köll and interpolated from Roadworks Drawings

Level A.O.D.	Dist from M'Way ℄
341.23	131.32
	90.45
320.65	80.45
320.98	70.45
	45'-6"
321.21	6.50
	0
321.21	6.50
319.47	77.78
319.14	87.78
319.14	97.78
341.00	141.50



Levels.

ch. 58+00	Interp. ch 58+65	ch. 59+20	Position
342.2	341.23	340.4	Top. N. Bank
339.6	341.00	342.2	Top. S. Bank.

ch. 58+50	Interp ch. 58+65	ch. 58+75	
321.66	321.66	321.65	N. Harg. Strip
321.19	321.21	321.24	Grade Line
	320.29		S. Harg. Strip

Span Lengths

a) Long Span

from diagram: OQ = 77.78' (O/A carriageway to ℄ at this point)

Hard Shoulder	=	9.50'	
Hard shoulder to Column face	=	5.00'	(M.O.T. minimum clearance)
Half Thickness of col.	=	1.50'	
		93.78	

We shall therefore make span L = 94'-0" long.

b) Short Span

from diagram, OQ − OP = 77.78 − 70.45
= 7.33 ft

∴ Span S = 93.78 − 7.33 = 84.45'

But Span L will have 16 bays of railings @ 5'-10½"
Then for span S we have 15 bays (say) @ 5'-10½" = 88'-1½"

Then Span S = 88'-1½" long.

Span Lengths (cont'd)

c) Shore Spans

Ht. of Embankments

(a) N Side = 341.23 − 320.65 = 20.58 ft.

b) S. Side : 341.00 − 319.14 = 21.86 ft.

Suppose we Increase long. fall of Bridge by 1.29 ft to 1.51 ft. (for Drainage purposes), then heights of abutments above H.S. level will be the same.

Assume Embankment Slope of 1:2.

Then length of Shore span will be 2 x 21.86 = 43.72 ft.

However, we require to make the span some convenient multiple of the coping module (2 per railing bay).

From inspection of a preliminary sketch 7½ railing bays = 44'-0¾" is suggested. This suits both shore spans and gives an acceptable size to the bench seat abutment.

Height Clearances

min clearance occurs at central reserve:-

G.L. level = 321.21 ft

bridge surface = $\frac{342.51 + 341.00}{2}$ = 341.76 ft.

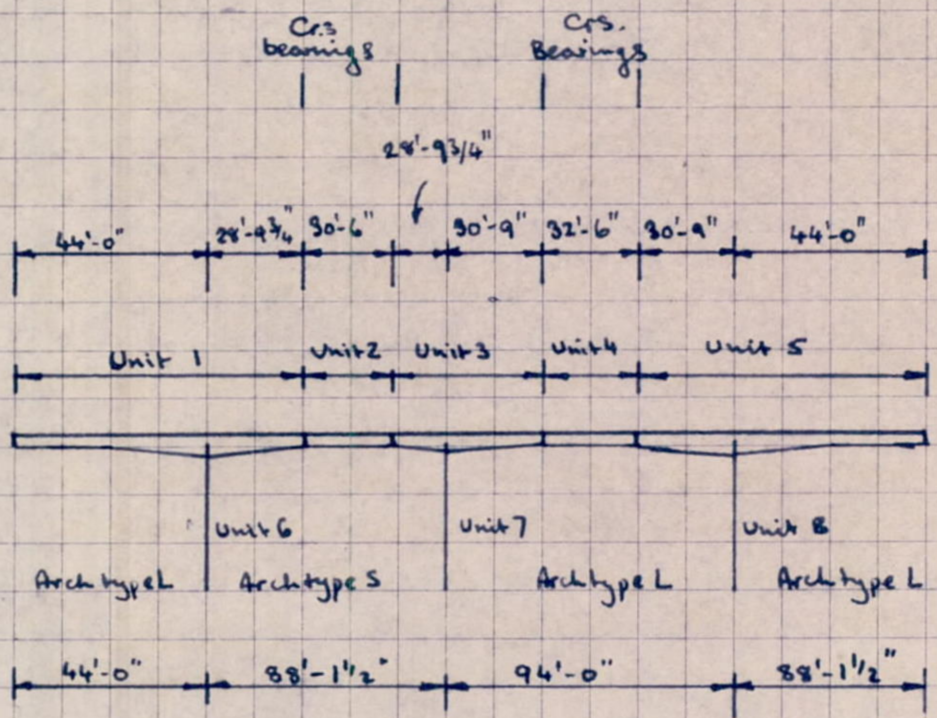
clearance = 341.76 − 321.21 = 20.55 ft.

minimum allowable clearance = 16.50 ft.

Thus allowable construction depth = 4.05 ft.

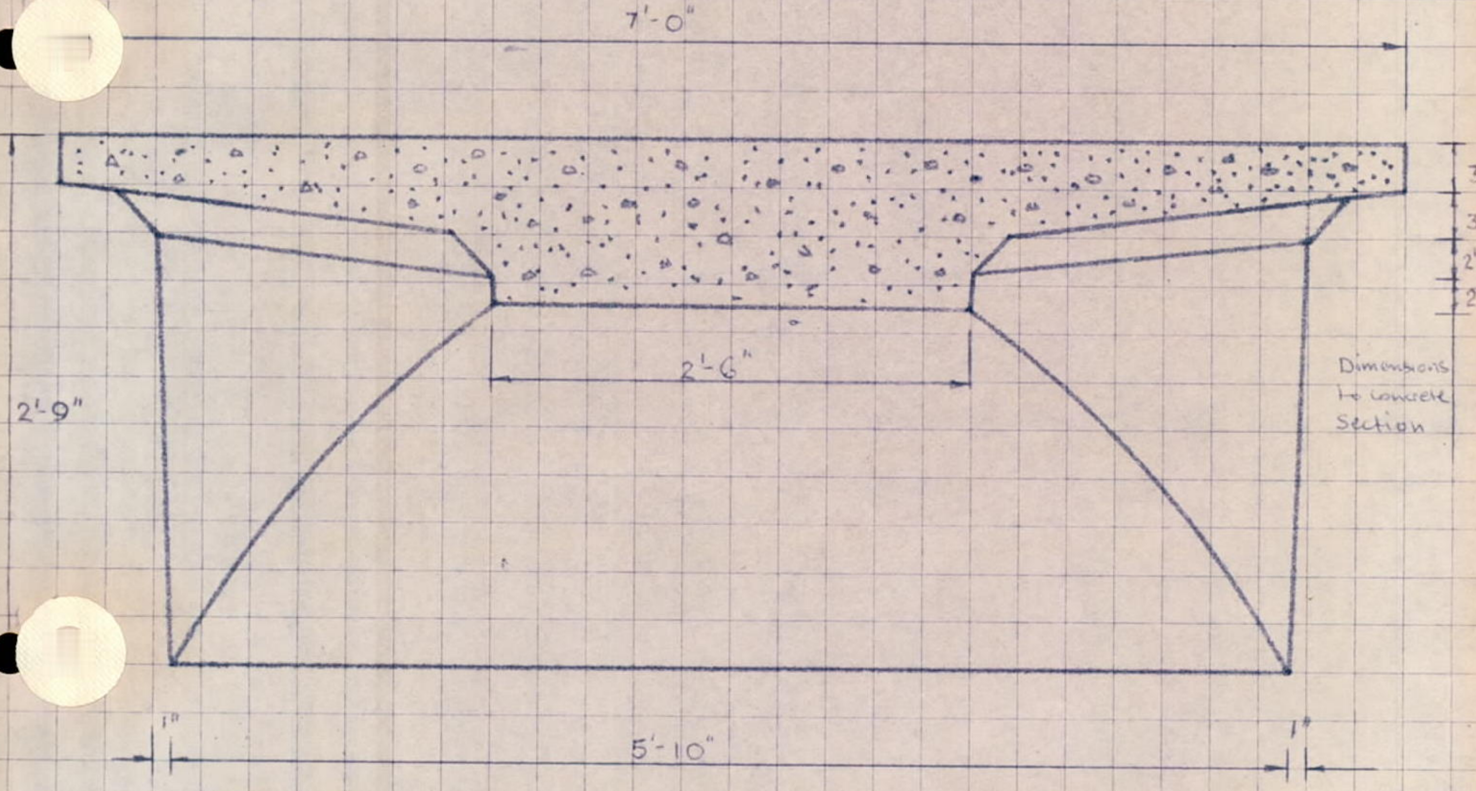
Since preliminary calculations suggest a construction depth of 3'-0" max. Clearances for the bridge are O.K.

From Preliminary Calculations, it has been decided to make the bridge of a suspended span-and-cantilever type of the following dimensions:-

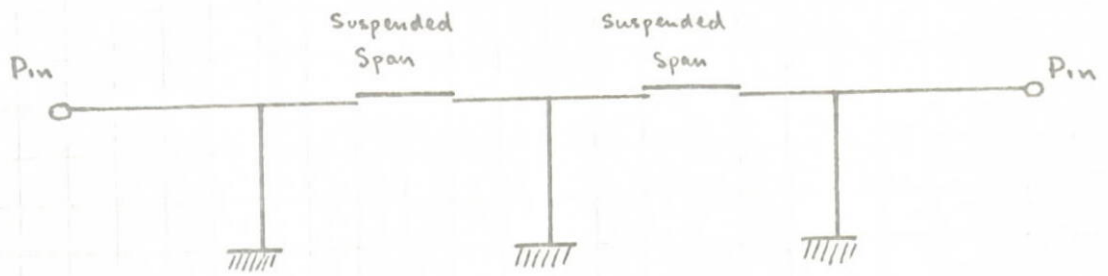


The form of the bridge will be of a spine beam varying in depth and width to parabolic (equal to circular) curves, supporting a homogeneous deck slab. It is anticipated that the parts of the bridge shown above will be precast, pretensioned members, i.e Units 1-5, and that units 6-8 (columns) will be steel box sections, tapering linearly in both elevations from top to bottom.

TYPICAL SECTION AT ARCH CROWN



Structural Diagram



In the preliminary design stage it was appreciated that the form of foot bridge envisaged, with very light sections and of suspended span and cantilever construction would be susceptible to unpopular and possibly deleterious vibrations. After consultation with the R.R.L., various Research pamphlets, and after visiting other bridges of a similar construction that have been built, the following measures were taken:

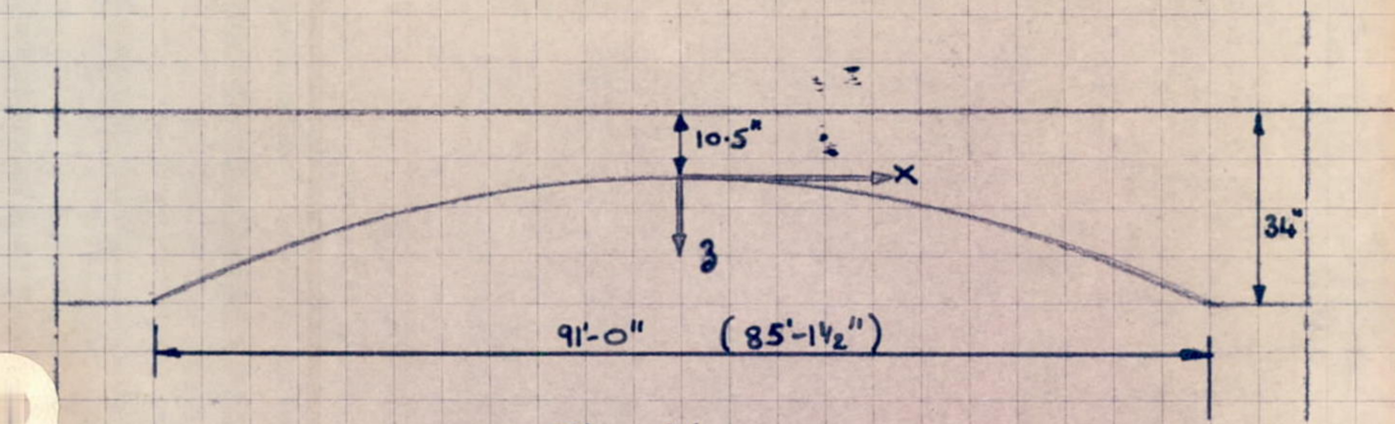
a) Pin the end spans to the abutments as shown. Increasing the fixity of a structure increases its rigidity.

b) The railings are made much stiffer in the longitudinal direction than is necessary to resist pedestrian loading only.

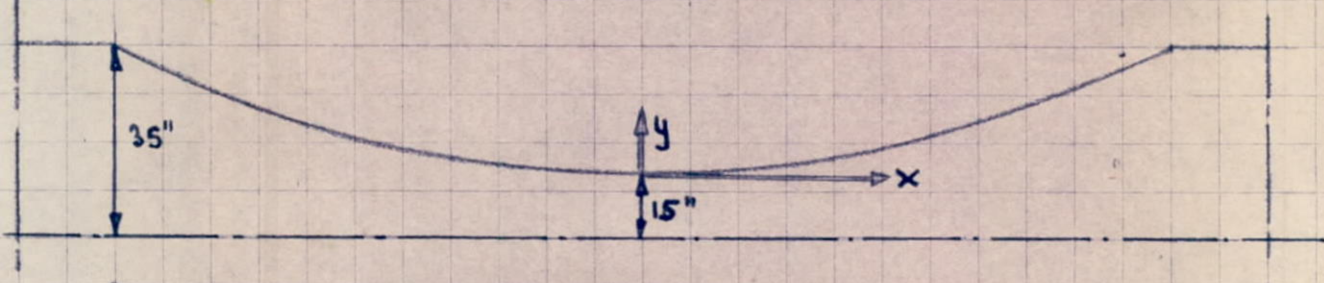
c) The mountings for the Railings are made ~~of~~ to a flexible, shock absorbing design.

It has proved impossible in the light of present knowledge to estimate either the amplitude or frequency of vibrations in the finished bridge.

Geometry of each Span



Elevation



Plan

Equation of Curves in Elevation

Curve L(ong) $\quad z = a x^2 \quad$ at $\left(45.5,\ \frac{23.5}{12}\right)$

$a = \dfrac{23.5}{45.5^2 \times 12 \times 12} = 0.0000078828$ (inch units)

Curve Short $\quad a = \dfrac{23.5}{(510.75)^2} = 0.0000900835$ "

Equation of Curves in Plan

Curve Long $\quad a = \dfrac{20}{(546)^2} = 0.000067088$ ~~0.0000060068~~

Curve Short $\quad a = \dfrac{20}{(510.75)^2} = 0.000076668$

Summary

Long Span (a) Curve LV105 $z = 0.000078828\,x^2$

(b) Curve LH105 $y = 0.000067088\,x^2$

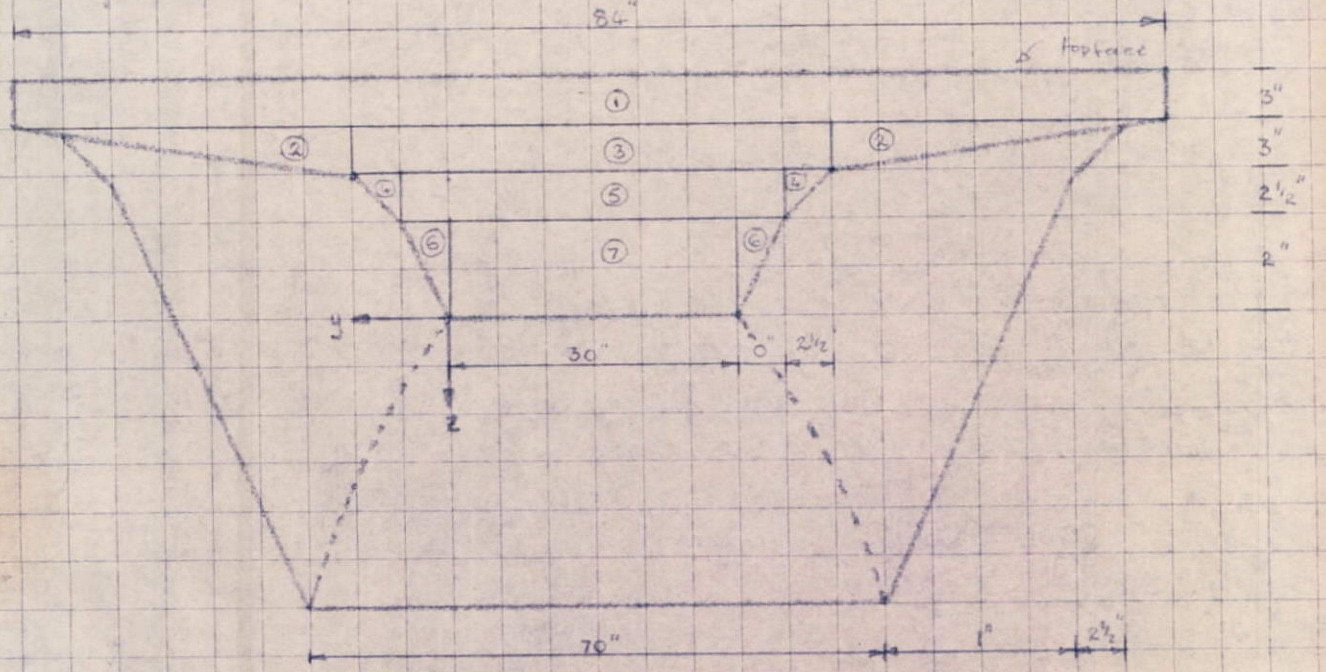
Short Span (c) Curve SV105 $z = 0.000090085\,x^2$

(d) Curve SH105 $y = 0.000076668\,x^2$

Co-ords at 2'-0" intervals

			Curve L (inches)			Curve S (inches)		
x ft	x ins	x^2 ins	z	y	Depth of Section	z	y	Depth of Section
0	0	0	0	0	10.50	0	0	10.50
2	24	576	0.05	0.04	10.55	0.05	0.04	10.55
4	48	2304	0.18	0.15 ~~1.57~~	10.68	0.21	0.18	10.71
6	72	5184	0.41	0.35 ~~3.48~~	10.91	0.47	0.39	10.97
8	96	9216	0.73	0.61 ~~6.18~~	11.23	0.83	0.71	11.33
10	120	14400	1.14	0.97 ~~9.66~~	11.64	1.29	1.10	11.79
12	144	19,600	1.54?	1.31?	12.04?	1.77?	1.50?	12.27?
14	168	28,224	2.22	1.89	12.72	2.54	2.16	13.04
16	192	36,864	2.90	2.47	13.40	3.32	2.83	13.82
18	216	46,656	3.68	3.13	14.18	4.20	3.58	14.70
20	240	57,600	4.54	3.86	15.04	5.19	4.42	15.69
22	264	69,696	5.49	4.68	15.99	6.28	5.34	16.78
24	288	82,944	6.54	5.56	17.04	7.47	6.36	17.97
26	312	97,344	7.67	6.53	18.17	8.77	7.46	19.27
28	336	112,896	8.90	7.57	19.40	10.17	8.65	20.67
30	360	129,600	10.22	8.69	20.72	11.68	9.94	22.18
32	384	147,456	11.62	9.89	22.12	13.28	11.31	23.78
34	408	166,464	13.12	11.17	23.62	14.99	12.76	25.49
36	432	186,624	14.71	12.52	25.21	16.81	14.31	27.31
38	456	207,936	16.39	13.95	26.89	18.73	15.94	29.23
40	480	230,400	18.16	15.46	28.66 ~~30.51~~	20.76	17.66	31.26
42	504	254,016	20.02	17.04	30.51 ~~32.48~~	22.88	19.47	33.38
44	528	278,784	21.98	18.70	32.48	—	—	—
45.5	546	298,116	23.5"	20.00	34.00	—	—	—
42.625	510.75	260,866	—	—	—	23.50	20.00	34.00

Analysis of Cross Sections for Computer Calculations of A $\bar{x}$ Z, Z_b + I



Item	A (breadth)	B (depth)	C (topface to top edge)	K (□ = 1, ◁ = 2)
1	84.0	3.0	0	1
2	$49.0 - 2y - \frac{2z}{23.5}$	$3 - \frac{3y}{24.5}$	3.0	2
3	$36.0 + 2y + \frac{2z}{23.5}$	$3 - \frac{3y}{24.5}$	3.0	1
4	5.0	2.5	$6 - \frac{3y}{24.5}$	2
5	$30 + 2y + \frac{2z}{23.5}$	2.5	$6 - \frac{3y}{24.5}$	1
6	$\frac{2z}{23.5}$	$2 + z + \frac{3y}{24.5}$	$8.5 - \frac{3y}{24.5}$	2
7	$30 + 2y$	$2 + z + \frac{3y}{24.5}$	$8.5 - \frac{3y}{24.5}$	1

A	Y	I	ZT	ZB	x (ft)
571.750	4.024	4643.463	1153.824	717.072	0
573.139	4.040	4700.185	1163.351	722.018	2
579.801	4.169	4946.811	1204.000	749.363	4
589.650	4.214	5341.288	1267.538	790.588	6
604.137	4.366	5935.656	1359.443	852.364	8
623.739	4.570	6782.047	1483.952	939.376	10
644.003	4.792	7719.034	1614.263	1030.822	12
679.114	5.143	9453.506	1839.219	1197.685	14
716.631	5.524	11491.762	2080.517	1385.136	16
761.933	5.968	14135.581	2368.458	1618.874	18
816.168	6.494	17584.016	2712.014	1910.008	20
879.899	7.056	22008.749	3114.794	2265.644	22
954.730	7.717	27672.954	3586.055	2698.964	24
1042.201	8.437	34931.742	4140.530	3224.431	26
1143.497	9.221	44156.826	4788.480	3856.987	28
1260.497	10.071	55908.401	5551.364	4617.137	30
1394.315	10.973	70696.994	6442.985	5520.058	32
1546.994	11.934	89443.909	7495.145	6597.900	34
1721.459	12.952	113228.784	8742.444	7885.901	36
1918.441	14.018	143168.284	10213.380	9411.361	38
2141.361	15.135	181004.572	11959.297	11225.126	40
2390.924	16.292	228308.496	14013.453	13360.823	42
2456.975	16.629	243787.186	14660.612	14033.920	42.5625

Susp. Span

Cantilever

Susp. Span

x = 15.25'

Cantilever

Curve S   All units in inches

Programme B.S.13.

Computer Print-Out

A = Cross Sectional Area

Y = Neutral axis — Top face

I = Second Moment of Area

X = Dist. from arch Crown (ft)

ZT = Section modulus (top face)

ZB = Section modulus (bottom face)

x(ft)	A	Y	I	ZT	ZB
0	571.750	4.024	4643.463	1153.824	717.072
2	573.734	4.044	4713.971	1165.581	724.592
4	578.468	4.096	4900.997	1196.583	744.360
6	587.579	4.190	5250.315	1252.962	781.334
8	600.007	4.323	5765.110	1333.510	834.708
10	616.930	4.500	6484.633	1440.986	908.229
12	633.981	4.678	7251.533	1550.107	985.006
14	664.232	4.991	8705.309	1744.183	1126.325
16	696.219	5.317	10362.369	1948.998	1281.959
18	734.847	5.703	12524.385	2196.168	1477.426
20	780.103	6.143	15257.659	2483.872	1714.862
22	833.489	6.643	18738.026	2820.590	2004.774
24	895.747	7.207	23168.783	3214.792	2356.211
26	967.747	7.827	28713.348	3668.532	2776.096
28	1056.208	8.486	35843.059	4223.861	3284.096
30	1146.901	9.248	44504.391	4812.303	3879.408
32	1255.847	10.037	55407.324	5520.254	4585.597
34	1380.326	10.893	69094.640	6348.980	5424.645
36	1521.093	11.776	86141.455	7314.811	6412.343
38	1679.882	12.717	107351.030	8441.258	7574.558
40	1858.725	13.702	133731.137	9759.736	8940.641
42	2058.537	14.730	166431.323	11298.795	10540.304
44	2284.056	15.796	207333.296	13125.516	12427.224
45.5	2466.975	16.629	243787.186	14660.612	14033.920

Susp. Span

x = 16.25'

Cantilever

Susp. Span

Cantile

Curve L

Programme B.S. 13.

Computer Print Out

Computer In-put

Effect of Jacking Pockets on X-section properties

M= 2

N	D
9	34.000
9	34.000

A	B	C	K	
84.000	3.000	0.000	1	
7.000	0.550	3.000	2	
77.000	0.550	3.000	1	
5.000	2.500	3.550	2	
72.000	2.500	3.550	1	
2.000	27.950	6.050	2	
70.000	27.950	6.050	1	
-25.000	4.000	0.000	1	4 Jacking pockets
-6.500	30.000	4.000	1	
84.000	3.000	0.000	1	
7.000	0.550	3.000	2	
77.000	0.550	3.000	1	
5.000	2.500	3.550	2	
72.000	2.500	3.550	1	
2.000	27.950	6.050	2	
70.000	27.950	6.050	1	
-21.000	4.000	0.000	1	3 Jacking pockets
-4.880	30.000	4.000	1	

Values of A, X, I, Zb, Zt, for Max beam section (over column) with Jacking pockets removed.

Computer Output

A	X	I	ZT	ZB	
2171.975	17.089	206071.588	12058.483	12185.902	4 pockets
2236.575	17.023	213548.485	12544.764	12578.632	3 pockets
2312.0			13104.0	13001	2 pockets (extrapolated)
2466.975	16.629	243,787.186	14660.612	14033.920	No pockets

May 9th 1967

Primary Calculations for Road Profile

1·03

Eq'n of Bridge Centreline

of form $y = m_1 x + c_1$ where $m_1 = \cot^{-1} 23^\circ 57' 34''$

$= 2.250323$

$y = 1190.25$

$x = 76023.58$

Then $c_1 = y - m_1 x = -76023.58 \times 2.250323 + 1190.25$

$c_1 = -\cancel{169885.612}$ -169887.3606

Bridge ℄ Eq'n. $\underline{\underline{y = m_1 x + c_1}}$ where $m_1 = 2.25032$

$c_1 = -\cancel{169885.612}$ 169887.3606

Eq'n of North Slip Road Margin

of form $y = m_2 x + c_2$

$$m_2 = \frac{1495.64 - 1242.10}{75390.16 - 76086.77} = -\cancel{0.3640}\ \cancel{-0.36396}\ -0.3639626$$

$$c_2 = 75390.16 \times 0.3640 + 1495.64 = \cancel{28934.7934}\ \cancel{28937.6582}\ 28934.8386$$

$\underline{\underline{y = m_2 x_2 + c_2}}$ where $m_2 = \cancel{-0.3640}$ -0.3639626

$c_2 = \cancel{28937.6582}$ 28934.8386

Eq'n of Chord of circular arc

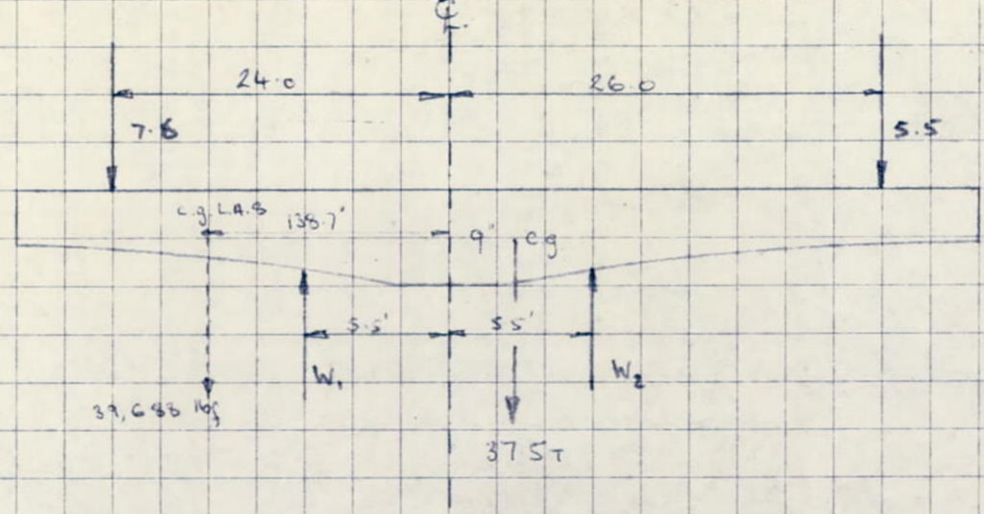
$$m_3 = \frac{1231.34 - 835.21}{75756.03 - 76498.87} = -\cancel{0.5333}\ 0.533264$$

$$c_3 = 75756.03 \times 0.5333 + 1231.34 = \cancel{41629.0}\ \cancel{41632.031}\ 41629.3036$$

$y = m_3 x + c_3$ where $m_3 = -0.5333$

$c_3 = 41632.031$

Unit 3



T.M.A. W_1

$$(7.6 \times 18.5) + (37.5 \times 6.25) = 5.5 \times 31.5 + W_2 \times 11.0$$

$$W_2 = \cancel{18.34}\ 24.28\ T$$

Vert. Eq'b'm.

$$W_1 = 7.6 + 5.5 + 37.5 - \cancel{18.34}^{24.28} = \cancel{32.26}\ 26.32\ T.$$

B.M. at ℄ (consider L.H.S)

$$M = (7.6 \times 24.0 - 26.32 \times 5.5)\,2240 \times 12 + 39,688 \times 138.7$$

$$= 6,516,488 \text{ lb.in.}$$

check $\sigma_{bot} = \frac{6,516,488}{14,033.92} - 810 + 431 = +85.3$ p.s.i.

By inspection σ_{top} of Unit 3, and σ_{bot} and σ_{top} of Unit 5 are safe.