SUSPENDED SPANS

The prestressed unit is designed and analysed in detail for stress conditions at 2ft. intervals. The Ultimate strength condition is checked and the deflection of the unit under Dead Load plus Superload evaluated.

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Prestressing Design of Suspended Span 33' 3½" long (32' 6" c/s bearings)

Section Properties at Centre (from computer print out)

$A = 572 \text{ in}^2$

$I = 4643 \text{ in}^4$

$Z_{top} = 1154 \text{ in}^3$

$Z_{bot} = 717 \text{ in}^3$

$\bar{x}_{top} = 4.02 \text{ in}$

$\bar{x}_{bot} = 6.48 \text{ in}$

Dead Load Moment $= 976,600$ lb.in

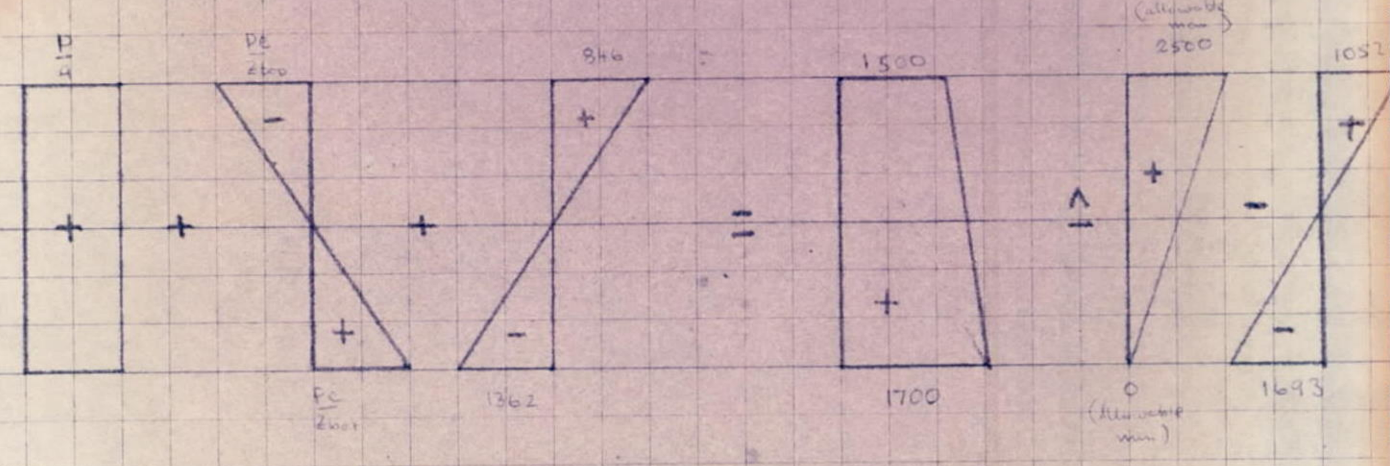
Live Load Moment $= \dfrac{776 \times (32.5)^2 \times 12}{8} = 1,213,760$ lb in

Dead Load Stresses $\sigma_{top} = \dfrac{976,600}{1154} = 846$ p.s.i.

$\sigma_{bot} = \dfrac{976,600}{717} = 1362$ p.s.i.

Live Load Stresses $\sigma_{top} = \dfrac{1,213,760}{1154} = 1052$ p.s.i.

$\sigma_{bot} = \quad = 1693$ p.s.i.



Equations $\dfrac{P}{A} - \dfrac{Pe}{Z_{top}} + 846 = 1500$ — ①

$\dfrac{P}{A} + \dfrac{Pe}{Z_{bot}} - 1362 = 1700$ — ②

$$\frac{P}{A} - \frac{Pe}{Z_{top}} = 654 \qquad -①$$

$$\frac{P}{A} + \frac{Pe}{Z_{bot}} = 3062 \qquad -②$$

① × Z_{top} + ② × Z_{bot}

$$\frac{P}{A}(Z_{top}) + \frac{P}{A}(Z_{bot}) = 654(Z_{top}) + 3062(Z_{bot})$$

$$\frac{P(Z_{top} + Z_{bot})}{A} = 654(Z_{top}) + 3062(Z_{bot})$$

Sub for Z & simplify

$$P = \frac{(654 \times 1154 + 3062 \times 717)\,572}{1154 + 717} \qquad -③$$

$$P = \underline{901,922 \text{ lbf}}$$

To find E

① − ② & sub for P $\quad e \times 901,922\left[\frac{1}{Z_{top}} + \frac{1}{Z_{bot}}\right] = 3062 - 654 \qquad ④$

$$e = \frac{2408}{781.6 + 1258} = \underline{1.18}$$

Transfer

$$P_t = \frac{901,922}{0.78} = 1156,310 \text{ lbf}$$

$$\frac{P}{A} = 2021 \text{ psi}$$

$$\sigma_{bot} = 2021 + 1903 - 1362 = \underline{2562 \text{ p.s.i. C}}$$

$$\frac{Pe}{Z_{bot}} = 1903$$

However, we have an end condition to satisfy. Since $\bar{x}_{top}$ at $x = 16.5'$ ≙ 5.4", we require an e ≙ 5.4 − 4.02 = 1.38, say 1.4", to avoid complications at the ends of the beam.

Sub in (2) for e $P = \dfrac{3062}{\left[\frac{1}{A} + \frac{e}{Z_{bot}}\right]} = \dfrac{3062}{0.001248 + .001453}$

$= .003701$

$P = \underline{\underline{827,350 \text{ lbf}}}$

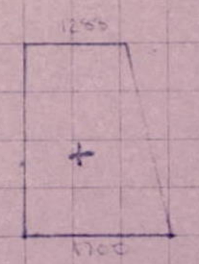
Sub in (1) for P + e

$\sigma_{top} = 1446 - 1004 + 846$

$= 442 + 846$

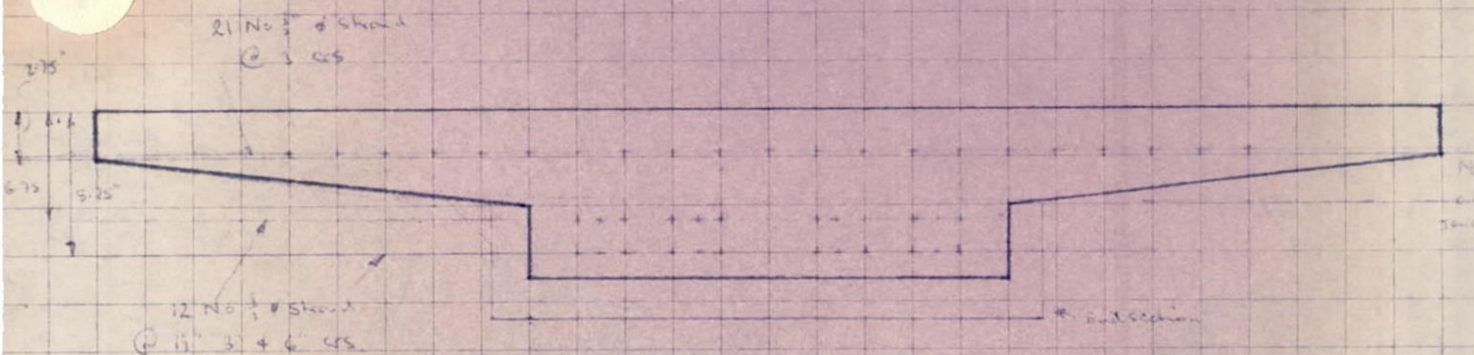
$= 1288$ psi

Then final diagram



$P_{initial} = \dfrac{827,350}{0.72} = 1,149,000$ lbf

Then we require $\dfrac{1,149,000}{25,900} = 45$ strands $\frac{1}{2}$" ⌀



Distribution.

Two rows of 12 in bottom
One row of 21 in top

(1) $21 \times 2 + 12 \times x + 12(x+2) = 45 \times 5.42$

$12x + 12x = 199.90$

$x =$

(2) $21 \times 2.5 + 12 \times x + 12x + 24 = 45 \times 5.42$

$24x =$

$x = 6.47$

(3) $21 \times 2.75 + 24x = 45 \times 5.42 - 24$ ✓

$x = 6.75\ (6.)$ in.

Recalculation of Transfer Stresses

$P = 827,350$ lbf

Then $\therefore P_t = \dfrac{827,350}{0.78} = 1,060,705$ lbf

$\dfrac{P}{A} = 1854$ p.s.i.

$\dfrac{Pe}{Z_{top}} = 1287$

$\dfrac{Pe}{Z_{bot}} = 2071$

Then $\sigma_{top} = 1854 - 1287 + 846 = \underline{1413}$ p.s.i. ✓

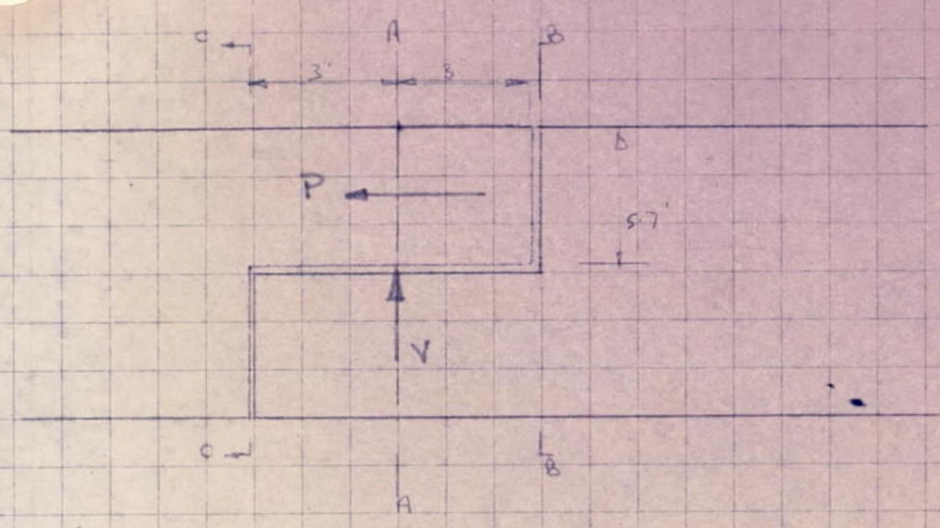
$\sigma_{bot} = 1854 - 2071 - 1362 = \underline{2563}$ p.s.i. ✓

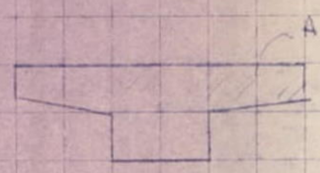
Check End Section

$A = 700$ in^2

$\dfrac{P_t}{A} = \dfrac{1060,705}{700} = \underline{1515 \text{ p.s.i.}}$ ✓ $\left(\dfrac{Pe}{Z} \text{ Or } \therefore e = 0 \right)$

Shear Stresses in Expansion Joint





$A \triangleq 84 \times 5.7 - 26 \times \dfrac{3.7}{2} = \underline{443.7 \text{ in}^2}$

Max Shear Force V = 10,351 + 766 × 16.25 = <u>22,800 lbf</u>

from C & CA research report No 11 transmission length of $\frac{1}{2}$" ⌀ strand is about 14" max with a roughly linear build up (graph No 2)

∴ Prestress force at AA = $\frac{3}{14.2} \times \frac{21}{45} \times 827,350$ [illegible]

$= \underline{82,735 \text{ lbf}}$

allowable principle stress $= \frac{82,735}{2 \times 443.7} - \left[\sigma^2 + \left(\frac{82,735}{2 \times 443.7}\right)^2\right]^{\frac{1}{2}} = -160$

~~93.235~~ $- \left[\sigma^2 + 8506\right]^{\frac{1}{2}} = -253$

$\sigma^2 = 64009 - 8506$

$= 55,503$

$\sigma = 236$ p.s.i

Allowable shear force $= \frac{236 \times 72 \times \frac{72}{~~24~~} \times (5.7)^3}{72 \times 3.0 \times 15 \times 12} = 58,274$ lbf O.K

i.e. > 22,800 lbf

<u>Analyse Section ~~BB~~ CC</u>

Prestress $= \frac{6}{14} \times \frac{21}{45} \times 827,350 = 165,470.0$ lbf.

R.H.S of section $M_D = 10,351 \times 3 = 31,050$ lb in (say)

$M_L = 12,450 \times 3 = 37,200$ lb in (say)

<u>Percentage of Tendon Steel at X = 0</u>

Cl. 314, C.P. 115 requires $\frac{A_{st}}{b \times d} \times 100 > \frac{15}{f_u}$ where: $A_{st} = 6.48 \text{ in}^2$

$b = 30$ in

$d = 10.5$ in

$f_u = 110$ t/in²

$\frac{A_{st}}{b \times d} = \frac{6.48}{30 \times 10.5} = 2.06\% > \frac{15}{110} = 0.136$

— O.K

Estimate of Prestress Losses – (see C.P. 115)

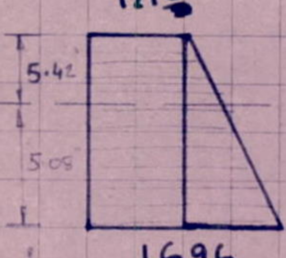
(a) **Steel creep.**

Stress relieved $\frac{1}{2}''\ \phi$ strand — **10,000 p.s.i**

(b) **Elastic deformation**

$$\frac{E_s}{E_c} = \frac{28 \times 10^6}{5.25 \times 10^6} = 5.35$$

Now σ_c, stress in adjacent concrete: ie at 5.42" from topface.



$$\sigma_c = 1296 + (1696 - 1296)\frac{5.42}{10.5} = 1502 \text{ p.s.i}$$

$$\therefore \text{loss} = \frac{E_s}{E_c} \times \sigma_c = 5.35 \times 1502 = \mathbf{8{,}036 \text{ p.s.i.}}$$

(c) **Concrete Shrinkage**

$$300 \times 10^{-6} \times 28 \times 10^{+6} = \mathbf{8{,}400 \text{ p.s.i}}$$

(d) **Concrete Creep.**

$$0.33 \times 10^{-6} \times 28 \times 10^{6} \times 1502 = \mathbf{13{,}878 \text{ p.s.i.}}$$

Total Losses — **40,314 p.s.i.**

Required Final Design Force = 827,350 lbf

Stress $= \frac{827{,}350}{0.144 \times 45} = 127{,}677$ p.s.i

Then $\sigma_{initial} = 127{,}677 + 40{,}314 = 167991$ p.s.i.

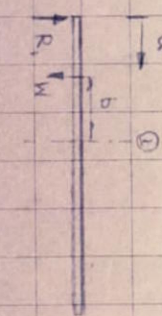
and $P_{initial} = 167{,}991 \times 0.144 \times 45 = 1{,}088{,}581$ lbf

and $P/_{strand}$ = **24,200 lbf**

$$\% \text{ Loss} = \frac{40{,}314}{167{,}991} \times 100 = 24\%$$

Stresses in Unit 4

Dead Load Moments



R_1 = 10,350 lb
W = Wt. of LHS
b = c.g. of LHS to ①

Bending Moment at ① = $R_1 a_1 - Wb$

Wt. at x ft.	Values of x (from Beam centre Line) ft: 0	1	3	5	7	9	11	13	15
0.5	3,576	—							
2.0	28,704	14,352	—						
4.0	57,888	43,416	14,472	—					
6.0	88,200	73,500	44,100	14,700	—				
8.0	120,000	105,000	75,000	45,000	15,000	—			
10.0	154,000	138,780	107,940	77,100	46,260	15,420	—		
12.0	190,000	174,240	142,560	110,880	79,200	47,520	15,840	—	
14.0	232,000	215,748	182,556	149,364	116,172	82,980	49,788	16,596	—
15.625	166,875	160,200	138,840	117,480	96,110	74,760	53,400	32,040	10,680
Wb lb.in.	1,041,900	925,236	705,468	514,524	352,752	220,680	119,028	48,636	10,680
$R_1 a_1$ lb.in.	2,018,250	1,894,050	1,645,650	1,397,250	1,148,850	900,450	652,050	403,650	155,250
B.M. lb.in.	**976,350**	**968,814**	**940,182**	**882,726**	**796,098**	**679,770**	**533,022**	**355,014**	**144,570**

Δ	0	28,632	57,456	86,628	116,328	146,748	178,008	210,444
Δ^2	-28,632	-28,824	-29,172	-29,700	-30,420	-31,260	-32,436	
Δ^3	0	192	348	528	720	840	1176	
Δ^4	-192	-156	-180	-192	-120	-336		
Δ^5	Numerical instability is too great for Δ^5 and Δ^6							
Δ^6								

Interpolation for B.M. at even values of x (Newtons forward Difference method)

Formula:-

$$f(x_0 + ph) = f_0 + p\Delta f_0 + \frac{p(p-1)}{2!}\Delta^2 f_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 f_0 + \ldots\ldots$$

here, $h = 2$

$x_0 = -15$ (initially)

we require $(x_0 + ph) = -14$ "

whence $p = 1/2$

Then $M_{14} = \text{~~210,444~~ } 144{,}570 + \frac{1}{2} . 210{,}444 + \frac{\frac{1}{2}.\left(-\frac{1}{2}\right)}{2!}(-32{,}436) +$

$+ \frac{\frac{1}{2}.(-1/2)(-3/2)}{3!} . 1176 + \ldots$

$= \underline{\underline{253{,}920 \text{ lb.in}}}$

$M_{12} = 355{,}014 + \frac{1}{2} . 178{,}008 + \frac{1}{8} . 31{,}260 + \frac{1}{16} . 840 + \ldots$
$= \underline{\underline{447{,}980 \text{ lb.in}}}$

$M_{10} = 533{,}022 + \frac{1}{2} . 146{,}748 + \frac{1}{8} . 30{,}420 + \frac{1}{16} . 720 + \ldots$
$= \underline{\underline{610{,}244 \text{ lb.in}}}$

$M_8 = 679{,}000 + \frac{1}{2} . 116{,}328 + \frac{1}{8} . 29{,}700 + \frac{1}{16} . 528 + \ldots$
$= \underline{\underline{740{,}910 \text{ lb.in}}}$

$M_6 = 796{,}098 + \frac{1}{2} . 86{,}628 + \frac{1}{8} . 29{,}172 + \frac{1}{16} . 348 + \ldots$
$= \underline{\underline{843{,}081 \text{ lb.in}}}$

$M_4 = 882{,}726 + \frac{1}{2} . 57{,}456 + \frac{1}{8} . 28{,}824 + \frac{1}{16} . 192 + \ldots$
$= \underline{\underline{915{,}069 \text{ lb.in}}}$

$M_2 = 940{,}182 + \frac{1}{2} . 28{,}632 + \frac{1}{8} . 28{,}632 + 0 + \ldots$
$= \underline{\underline{958{,}077 \text{ lb.in}}}$

Dead Load Stresses

$$\sigma_{top} = \frac{M}{Z_{top}} \qquad \sigma_{bot} = \frac{M}{Z_{bot.}}$$

x ft.	Calculated B.M. lb.in	Interpolated B.M. lb.in.	σ_{top} p.s.i	σ_{bot} p.s.i.
0	976,350	976,350	+ 846	− 1362
1	968,814			
2		958,077	+ 822	− 1322
3	940,182			
4		915,069	+ 765	− 1229
5	882,726			
6		843,081	+ ~~597~~ 673	− 1079
7	796,098			
8		740,910	+ 556	− 888
9	679,770			
10		610,244	+ 423	− 672
11	533,022			
12		447,980	+ 289	− 455
13	355,014			
14		253,920	+ 146	− 225
15	144,570			

Prestressing Stresses

Final prestressing force, P = 827, 350 lbf.

Centroid of tendons to top face = 5·42 ins

∴ Eccentricity, e = 5·42 − Y (see computer print out)

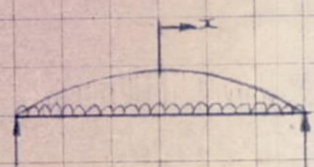
Prestressing Stresses (continued)

x ft.	$e = 5.42 - Y$ ins	$\frac{Pe}{Z_{top}} = \sigma_t$ ins	$\frac{Pe}{Z_{bot}} = \sigma_b$ ins	$\frac{P}{A}$ ins
0	1.396	-1,001	+1611	+1447
2	1.376	-977	+1571	+1442
4	1.324	-915	+1472	+1430
6	1.230	-812	+1302	+1408
8	1.097	-681	+1087	+1379
10	0.920	-528	+838	+1341
12	0.742	-396	+623	+1305
14	0.429	-203	+315	+1246

Liveload Stresses

Live load is uniform u.d.l of. 766 lb/ft run.
Span 32.5 ft.

Bending Moment equation :-



$$M_x = M_{max} - \frac{w x^2}{2}$$

where $M_{max} = \frac{766 \times 32.5^2 \times 12}{8} = 1,213,700$ lb.in

$w = 766$ lb/ft run

Liveload Moments and Stresses

$$M_x = 1,213,700 - \frac{766}{2} \times x^2$$

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x ft	Bending Moment lb.in	σ_{top} p.s.i	σ_{bot} p.s.i.
0	1,213,700	+ 1052	− 1693
2	1,195,316	+ 1026	− 1650
4	1,140,164	+ 953	− 1532
6	1,048,244	+ 837	− 1342
8	919,556	+ 690	− 1102
10	754,100	+ 523	− 830
12	551,876	+ 356	− 560
14	312,884	+ 178	− 278

STRESSES FOR SUSPENDED SPAN ~ UNIT 4 (Long)

Table E ~ σ_{top} p.s.i.

Col No.	①	②	③	④	⑤	⑥
x ft	Dead Load	$\frac{Pe}{Z_{top}}$	$\frac{P}{A}$	1 + 2 + 3	Live Load	4 + 5
0	+846	−1,001	+1447	**+1292**	+1052	**+2344**
2	+822	−977	+1442	**+1287**	+1026	**+2313**
4	+765	−915	+1430	**+1280**	+953	**+2233**
6	+673	−812	+1408	**+1269**	+837	**+2106**
8	+556	−681	+1379	**+1254**	+690	**+1944**
10	+423	−528	+1341	**+1236**	+523	**+1759**
12	+289	−346	+1305	**+1198**	+356	**+1554**
14	+146	−203	+1246	**+1189**	+178	**+1367**

Table F ~ $\sigma_{bot.}$ p.s.i.

Col No.	①	②	③	④	⑤	⑥
x ft	Dead Load	$\frac{Pe}{Z_{bot}}$	$\frac{P}{A}$	1 + 2 + 3	Live Load	4 + 5
0	−1362	+1611	+1447	**+1696**	−1693	**+3**
2	−1322	+1571	+1442	**+1691**	−1650	**+41**
4	−1229	+1472	+1430	**+1673**	−1532	**+141**
6	−1079	+1302	+1408	**+1631**	−1342	**+289**
8	−888	+1087	+1379	**+1578**	−1102	**+476**
10	−672	+838	+1341	**+1507**	−830	**+677**
12	−455	+623	+1305	**+1473**	−560	**+913**
14	−225	+315	+1246	**+1336**	−278	**+1058**

Ultimate Strength of Unit 4, Suspended Span 32'-6" long

Load Factors (C.P.115 : 1959)

$M_u \geq 2(M_D + M_L)$ where M_U = Ultimate moment of resistance
M_D = Dead Load Moment
M_L = Live Load Moment

Then $M_u > 2(976,600 + 1,213,700) =$ **4,380,600 lb.in**

For the analysis of this unit, the method as laid down in C.P. 115 is unsuitable, and the method as shown in CCL prestressed Design booklet will be used, treating the unit as a 'T' beam.

Equation:-

$$M_u = K \times A_{stw} \times f_u \times d_1 \left[1 - 0.75 \frac{A_{stw} \times f_u}{b' \times d_1 \times U_w}\right] + 0.7\, U_w \times t(b-b')(d_1 - 0.5t)$$

Where: $K = 1$ (coefficient of bond efficiency)
$f_u = 246,000$ lb/in² (ultimate tendon stress)
$d_1 = 5.54$ ins (depth from top face to tendon centroid)
$b' = 30$ ins (breadth of web)
$b = 72$ ins (width of flange)
$t = 4$ ins (Average thickness of flange)
$U_w = 7,500$ p.s.i (28 day concrete strength)
$A_{stw} = A_{st} - A_{stf}$
$A_{st} = 6.48$ in² (Area of steel tendons)
$A_{stf} = 0.68\, U_w (b-b') \dfrac{t}{f_u}$

$$A_{stf} = 0.68 \times 7,500\,(72-30) \cdot \frac{4}{246,000} = 3.48 \text{ ins}$$

Then $A_{stw} = 6.48 - 3.48 = 3$ ins

Then:-

$$M_u = 3 \times 246,000 \times 5.54 \left[1 - \frac{0.75 \times 3 \times 246,000}{30 \times 5.54 \times 7,500}\right] + 0.7 \times 7,500 \times 4\,(42)(5.54 - 2)$$

$= 2,289,600 + 3,122,300$ lb.in

M_u = 5,411,900 lb.in — O.K

Design of Suspended span 31'-4" long (30'-6" c/s bearings)

Section properties at centre (as for ~~other~~ long beam)

$A = 572 \text{ in}^2$

$I = 4443 \text{ in}^4$

$Z_{top} = 1104 \text{ in}^3$

$Z_{bot} = 717 \text{ in}^3$

$\bar{x}_{top} = 4.02 \text{ in}$

$\bar{x}_{bot} = 6.48 \text{ in}$

Dead Load Moment = 858,135 lb in

Live Load Moment = $766 \times \frac{30.5^2}{8} \times 12$ = 1,068,960 lb. in.

Dead Load Stresses σ_{top} = 926 p.s.i.
σ_{bot} = 1197 p.s.i.

Live Load Stresses σ_{top} = 926 p.s.i.
σ_{bot} = 1491 p.s.i.

We shall try to keep same wire formation, then let e = 1.40 in.

As before sub in (2)

$$\frac{P}{A} + \frac{Pe}{Z_{bot}} - 1197 = 1500$$

$$P = \frac{2,697}{\frac{1}{572} + \frac{1.4}{717}} = \frac{2,697}{0.003701} = \underline{728,730 \text{ lbf}}$$

we can either (a) use 45 No ½" d strand at $\frac{728,730}{45 \times 0.72}$ = ~~16~~, 22,490 lbf

or (b) use $\frac{728,730}{25,900 \times .72}$ = <u>40 No strand</u>

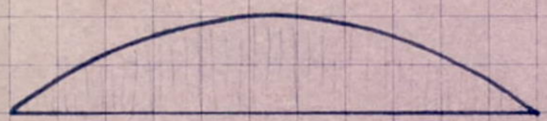
[We have shown in the analysis of Unit 4 that the central section is critical. Therefore No further analysis will be made of unit 2]

(a) After reestimation of losses, P/strand = $\frac{728,730}{45 \times 0.76}$ = 21,400 lbf (say)

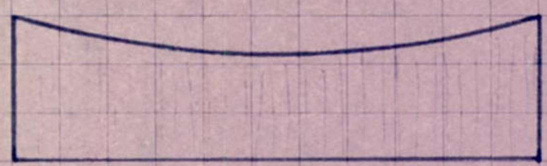
Deflection of Unit 4 under Dead Load and Prestress and Superload

Using Mohr's Theorems

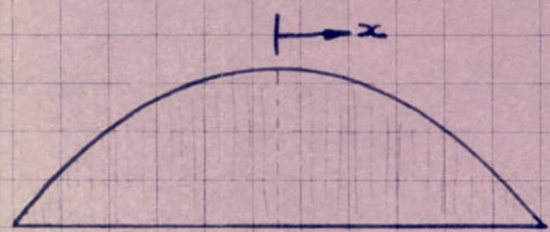
M diag.



e diag.



$\frac{M}{I}$ diagram.



We require moment of $\frac{1}{2} \cdot \frac{M}{EI}$ diagram about one end. The diagram will be considered as made up of strips, generally 2 ft wide, centred about even values of x.

P = 827,350

x ft	Dead Load B.M. M_D lb in	e ins	P e lb in	$M_D - Pe$ lb in	Superload Moment M_s lb in	$M_D - Pe + M_s$ lb in
0	976,350	1·396	1,154,980	−178,630	263,021	84,391
2	958,077	1·376	1,138,433	−180,356	259,037	78,681
4	915,069	1·324	1,095,411	−180,342	247,085	66,743
6	843,081	1·230	1,017,640	−174,559	227,165	52,606
8	740,910	1·097	907,602	−166,692	199,277	32,585
10	610,244	0·920	761,162	−150,918	163,421	12,503
12	447,980	0·742	613,893	−166,613	119,597	−47,016
14	253,920	0·429	354,933	−101,013	67,805	−33,208
16	2,400	0·003	2,482	− 82	—	—

x ft	$M_D - Pe + M_S$ = M lb.ins	I ins^4	$\frac{M}{I}$ lb/in^3	Strip width ins	Area of Strip lb/in^2	Arm in	Moment about End lb/in
0	84,391	4643	18.18	12"	218.16	189	41,232
2	78,681	4714	16.69	24	400.56	171	68,495
4	66,743	4901	13.62	24	326.88	147	48,051
6	52,606	5250	10.02	24	240.48	123	29,579
8	32,555	5765	5.65	24	135.60	99	13,424
10	12,503	6485	1.93	24	46.32	75	3,474
12	- 47,016	7252	-6.48	24	- 155.52	51	- 7,931
14	- 33,208	8705	- 3.81	24	- 91.44	27	- 2,469
16	-	10,362	0	9	0	3	0

$\Sigma M = 193,855$ lb/in

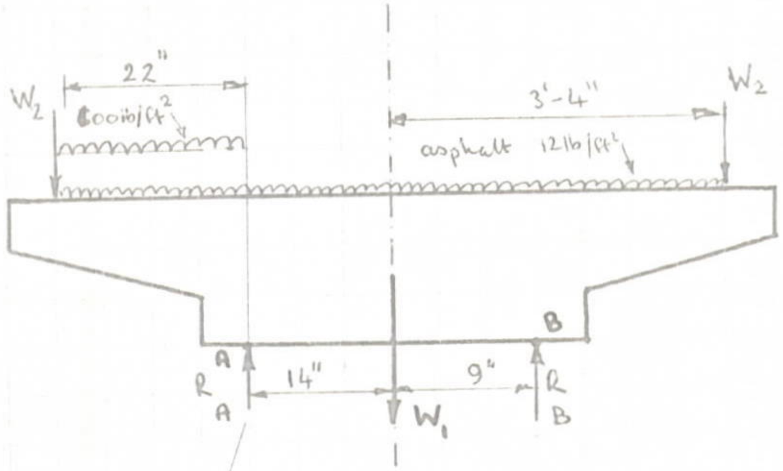
Deflection, $\delta = \frac{\Sigma M}{E_c} = \frac{193,855}{5.75 \times 10^6}$
(of centre relative to end)

$\delta = + 0.034''$ (downwards)

This deflection of $1/32''$ is small enough not to have to consider 'setting up' of formwork.

By examination of the $\frac{M}{I}$ diagrams, it can be seen that unit 2 will have a similar deflection, and no further action need be taken.

Lateral Stability of Suspended Span   32'-6" long.



	W_1 = Wt of Beam / 2 =	10,330 lbf.
((70+14)×16)2	= W_2 = coping + railing =	2,690
	Asphalt = 12×6×16 =	1,150
	Σ Load acting on Centreline =	14,190 lbf.

T.M.A. A.

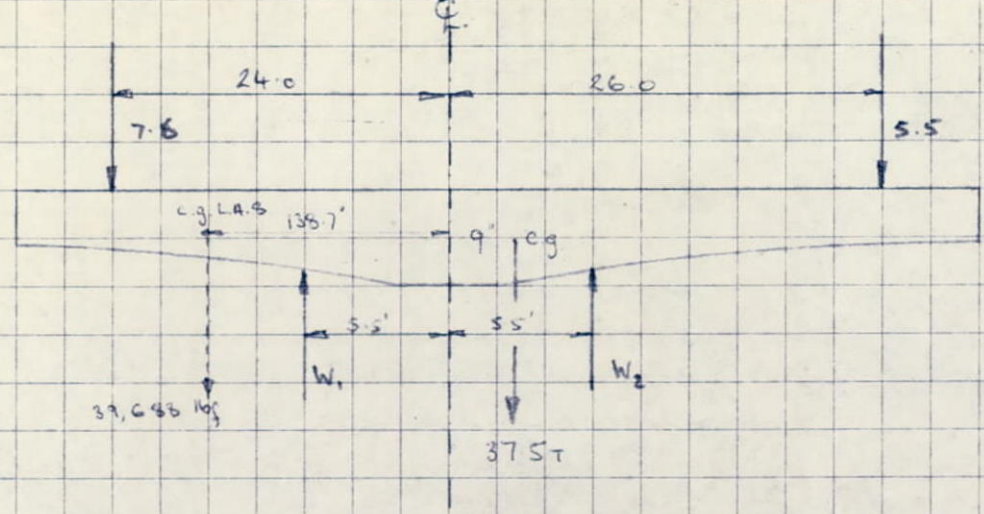
$$R_B \times 23 + 100 \times 2.16 \times 16.25 \times \frac{12}{2} = 14{,}190 \times 14$$

$$R_B = +\ 6{,}810 \text{ lbf.}$$

i.e. R_B is +ve. so the beam is stable.

(Torsional Moment, T = 6,810 × ~~9~~ 23 = ~~61,300~~ 154,000 lb.in)

Unit 3



T.M.A. W_1

$(7.6 \times 18.5) - (37.5 \times 6.25) = 5.5 \times 31.5 - W_2 \times 11.0$

$W_2 = $ ~~15.34~~ 24.28 T

Vert. Eq'b'm.

$W_1 = 7.6 + 5.5 + 37.5 - 24.28$ ~~16.34~~ $=$ ~~32.26~~ 26.32 T.

B.M. at ℄ (consider L.H.S)

$M = (7.6 \times 24.0 - 26.32 \times 5.5)\,2240 \times 12 + 39,688 \times 138.7$

$= 6,516,488$ lb.in.

check $\sigma_{bot} = \dfrac{6,516,488}{14,033.92} - 810 + 431 = +85.3$ p.s.i. (464)

By inspection σ_{top} of Unit 3, and σ_{bot} and σ_{top} of Unit 5 are safe.

[signature]