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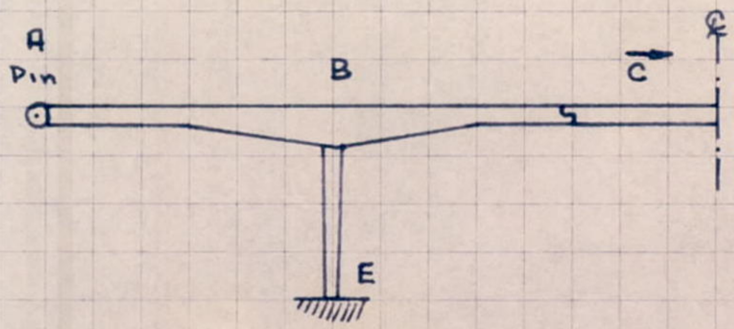
END FRAMES

A computer print-out for the statically indeterminate endframe (live load only), giving moment distribution factors is presented. Bending moment diagrams are constructed for various live load conditions. Detailed Dead load Bending moment diagram is drawn, and the end span stresses are analysed in detail. Stresses at various stages of construction are examined for the prestressed unit. Deflection under the influence of Dead Load only is calculated. The pin joint to the abutment is designed. Secondary stresses are investigated

CONTENTS **PAGES**

Design of End Frames.



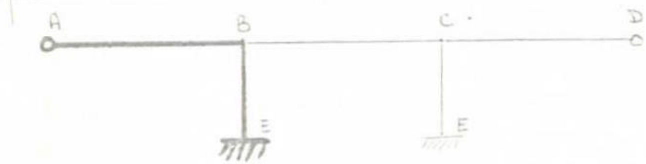
A computer programme has been prepared that calculates the moment distribution factors and ~~constants~~ coefficients for this end frame, using the cross section properties given elsewhere for the column and the beam units. The computer print out is overleaf, and the frame has a distribution thus:-

Column
Cantilever

	A (Pin)	AB	B	BC	E (Fixed)
D.Fs.	Pin	0.053	0.947	Cant	Fixed
C.O.Fs.	→ 1.434	0.234 ←	↑ 0.321		↑ 0.783

It is intended that Units 1 & 5 should have the same stressing arrangements as Unit 3. Any difficulties encountered towards the pin-end will be tackled by debonding tendons as necessary from that end.

CAB	CBA	CBC	CCB	[illegible]D	CDC	CEB	CBE	Carry Over Factors
-1.434	-0.234	-0.500	-0.500	[illegible]00	-0.500	-0.783	-0.[illegible]	
DBL	DBC	DBR	DCL	DCC	DCR	Distribution Factors		
0.053	0.947	0.000	0.000	1.000	0.000			



FBL	FBR	FCL	FCR	Fixed End DL Moments
-5019.74	-0.00	-0.00	-0.00	

MBC	MCC	Distributed DL Moments
-4752.17	0.00	

Bending Moments at Tenth Points of Spans

All Moments x 1000

MA	1	2	3	4	5	6	7	8	9	MBL
0.00	366.52	603.91	699.55	639.13	404.24	-33.03	-710.71	-1678.52	-3001.67	-4752.17
MBR	11	12	13	14	15	16	17	18	19	MCL
-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	0.00	-0.00
MCR	21	22	23	24	25	26	27	28	29	MD
-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	0.00	0.00

END FRAME DESIGN FACTORS

Using span L 10½

Programme B.S. 15

Live Load on BC of Unit 1

Cantilever Moment (from elsewhere) = 8,209,000 lb in

Live Load on AB of Unit 1 (+2)

FEM's AB, BA = $\frac{766 \times 43^2}{12} \times 12$ = 1,416,400 lb in

DF's BA : BE = ~~0.052 0.9~~ 0.053 : 0.947 } from computer printout B = 15

COF's CAB = 1.434, BE = 0.321

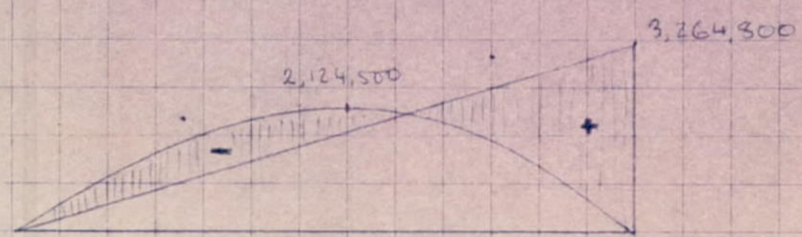
Moment Distribution

Column

1.434 →

A	B		(C)	E
0	0.053	0.947	Cantilever	Fixed
−1,416,400	+1,416,400	0		0
+1,416,400 →	+2,031,120			
	− 182,710	− 3264,800		− 1,048,000
0	+ 3264,800	− 3264,800		− 1,048,000

Max Simply Supported Moment AB = $\frac{766 \times 43^2}{8} \times 12$ = 2,124,500 lb in



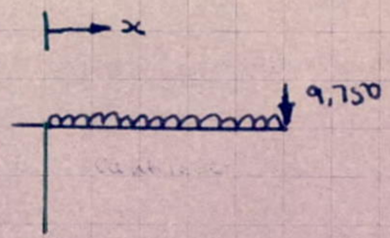
Bending Moment Equation :-

$$M_x = \frac{w.lx.}{2} - \frac{wx^2}{2} - M_F \frac{x}{l}$$

where: l = 43'-0"
w = 766 lb/ft run
M_F = 3,264,800 lb.in.

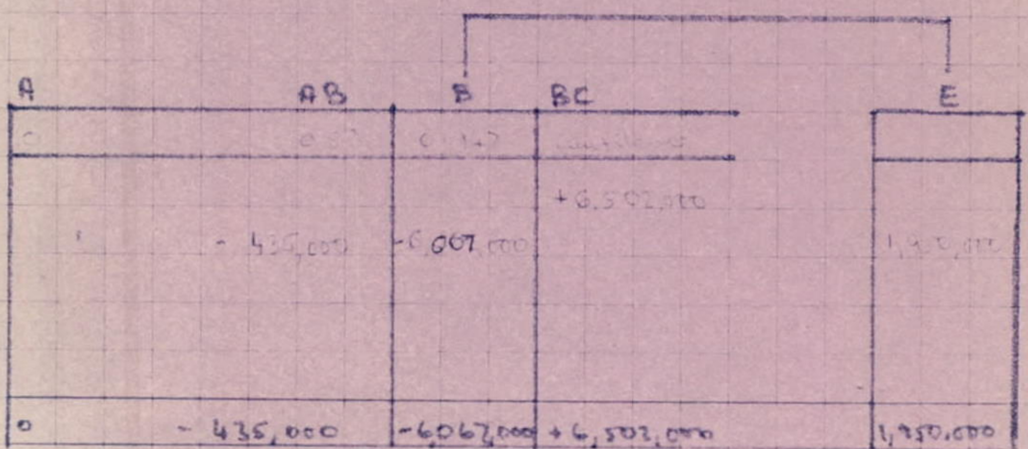
Live Load on BC only

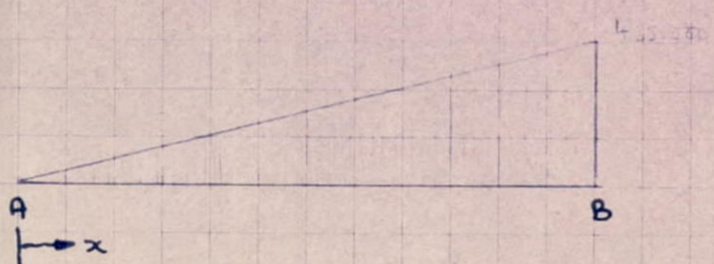
Bending Moment Equation :-



(see column Design)

$$M_x = 6,502,000 - 600 \cdot \frac{x^2}{2} - 9,750 \cdot x$$





Bending Moment Equation :

$$M_x = M_F \cdot \frac{x}{L}$$

where M_F = 435,000
L = 43'-0"

Temperature Stresses

Moment at Top of Column $= \frac{R_A EI}{h}(1 - C_A)\frac{\delta}{h}$ — (1)

where C $\frac{K \cdot I}{h}$ = [illegible]

To derive R_A – Handbook of Frame constants page 24

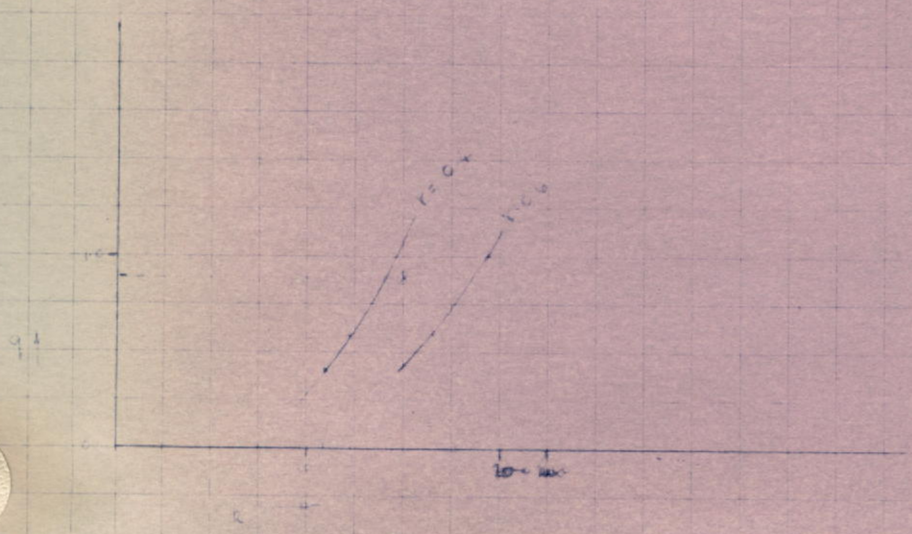
[illegible] 34

$q = \frac{[illegible] - [illegible]}{[illegible]}$ = 0·[illegible]

$h \cdot h = 70$

$r = \frac{70 - 45}{48} = 0·458$

Interpolation in table 55 gives k = 15·4



I = ~~430,000~~ 71,100 in^4 ([illegible])

t = 2 – [illegible] 5·75

h = [illegible] ?

C_A = 0·321

δ = 50 × 43 × 12 × 0·000006

= 0·16 in

$M = \frac{15·4 \times 2 \times [illegible] \times 431 \times 10^3}{20 \times 12 \times 10 \times 12}(1 - [-0·341]) \times 0·16$ = ~~8,445,660~~ 23,600,000 lb in

Distribution

	AB	B		E
	0·053	0·947		
		~~+ 8,445,660~~		
	– 431,830	– 7,716,790		
	– ~~431,830~~	+ ~~431,830~~		~~[illegible]~~
	– 1,242,000	+ 1,242,000		398,600

Shrinkage Stresses

We can assume that by the time the unit is ~~in position and~~ finally locked in position, it is at least 2 months old, by which time $\frac{2}{3}$ of shrinkage has occurred.

Shrinkage coefficient, $\alpha = \frac{1}{3} \times$ ~~600~~ 300×10^{-6}

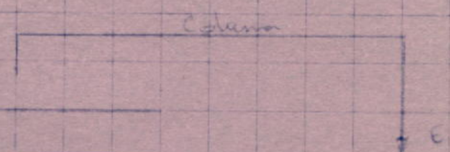
$\delta =$ ~~[illegible]~~ $\times 43 \times 12 \times 100 \times 10^{-6}$

Subst in eqn (1) $= 0.052$

$$M = \frac{15.4 \times 2 \times 10^6 \times 72.1 \times 10^3 \times 1.321 \times 0.052}{400 \times 44}$$

$M =$ ~~2,648,910~~ 7,613,400 lb in

Distribution



AB	B
0.053	0.947
	7,613,900
-403,536	-7,210,364
+403,536	+403,536

~~[illegible]~~ 124,535

Temperature Moments $= \pm$ 403,536

Shrinkage Moment

Calculation for position and value of Max B M for LL in AB

Equation of Parabola

$$M_1 = \frac{w}{2} l x - w x . \frac{x}{2} \qquad \text{where } w = 766, \quad l = 43$$

Equation of Straight line

$$M_2 = m x \qquad \text{where } m = \frac{3,264,800}{43 \times 12}$$

$$\text{Resultant M} \quad M_1 - M_2 = \frac{w l x}{2} - \frac{w x^2}{2} - m x \qquad \text{--- (3)}$$

for Max or Min

$$\frac{dM}{dx} = 0 = \frac{w l}{2} - w x - m = 0$$

$$x = \frac{l}{2} - \frac{m}{w}$$

Sub for l, m, w

$$x = \frac{43}{2} - \frac{3,264,800}{43 \times 766 \times 12}$$

Max B.M at :- $x = 13.24$ ft.

Max B.M, sub in (3)

$$M = \frac{766 \times 43 \times 13.24}{2} - \frac{766 \times (13.24)^2}{2} - \frac{3,264,800 \times 13.24}{12 \times 43}$$

$$= 67,134 \text{ lb ft}$$

$$M_{max} = 805,670 \text{ lb in}$$

And M = 0 at -

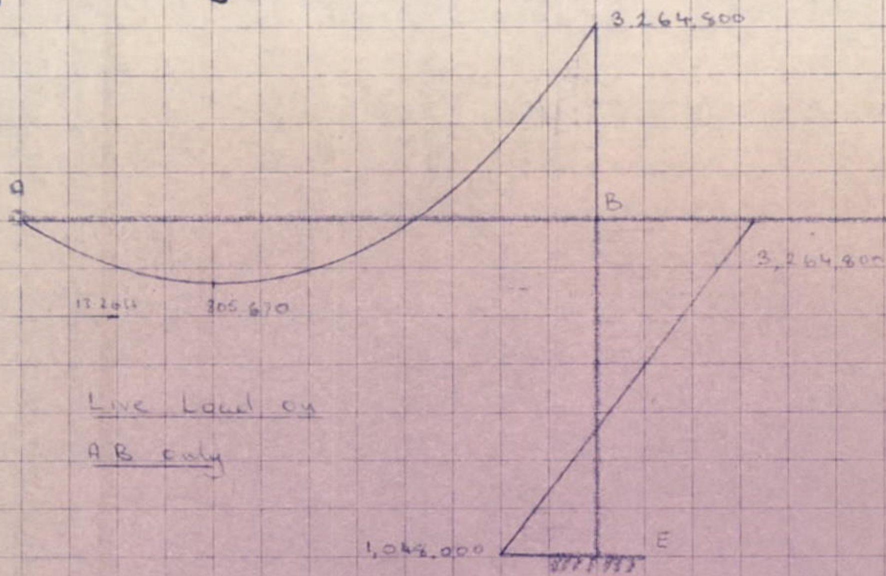
$$\frac{w}{2} l x - \frac{w x^2}{2} - m x = 0$$

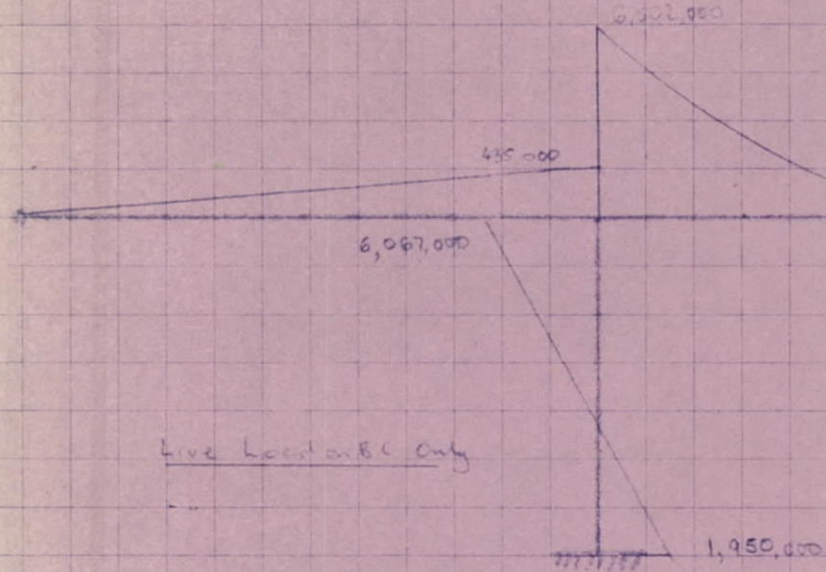
$$x = \left(\frac{w l}{2} - m\right) \frac{2}{w} = l - \frac{2m}{w}$$

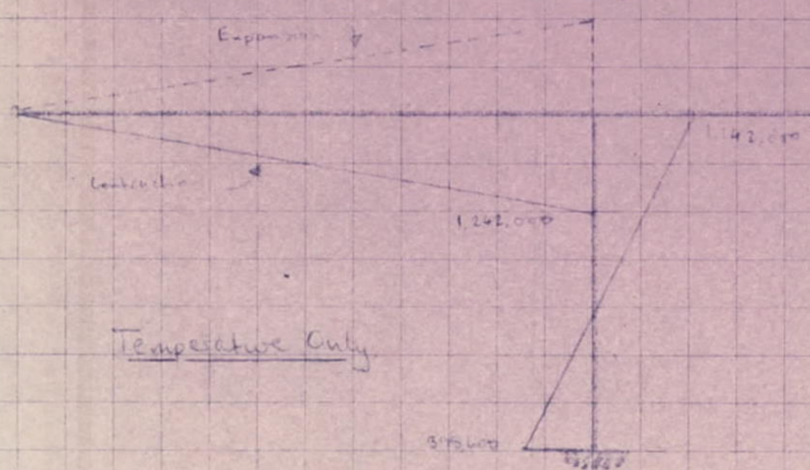
$$x = 43 - \frac{2 \times 3,264,800}{43 \times 766 \times 12}$$

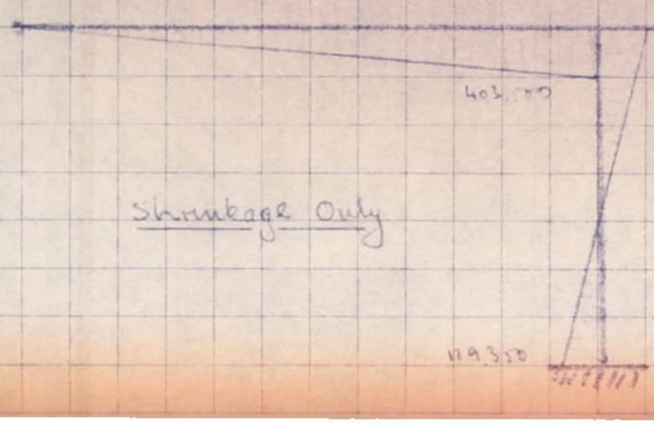
$x = 26.48$ ft.

Bending Moment Diagrams









Stresses in AB

Live load on AB only

x ft	Live Load M lb in	σ top lb/in²	σ bottom lb/in²
2.75	-30,135	25.564	-41.228
4.0	-172,213	143.921	-226.409
6.0	-369,657	245.027	-473.110
8.0	-530,334	397.698	-636.353
10.0	-654,243	454.025	-720.350
12.0	-741,384	478.279	-752.670
14.0	-791,697	453.907	-702.403
16.0	-805,362	413.218	-628.228
18.0	-782,199 ~~-791,697~~	356.165	-529.434
20.0	-722,267	290.783	-421.191
22.0	-625,568	221.786	-312.039
24.0	-492,101	153.674	-208.853
26.0	-321,866	87.737	-115.942
28.0	-114,863	27.194	-34.976
30.0	128,908	-26.787	33.229
32.0	409,448	-74.172	84.290
34.0	726,755	-114.468	133.973
36.0	1,080,830	-147.759	168.555
38.0	1,471,673	-174.343	194.292
40.0	1,849,284	-194.607	212.433
42.0	2,363,664	-209.196	224.250
44.0	2,864,811	-218.262	230.527
45.5	3,264,800	-222.691	232.636

Stresses in AB

Lineload on BC only

$$M = \frac{435,097}{43.0}(x - 2.5) \text{ lb-ins}$$

x ft	(x-2.5) ft	M lb-ins	σ top lb/in²	σ bottom lb/in²
2.75	0.25	2,524.634	-2.147	3.460
4.0	1.5	15,177.862	-12.684	20.390
6.0	3.5	35,414.812	-28.265	45.326
8.0	5.5	55,651.942	-41.733	66.672
10.0	7.5	75,887.012	-52.665	83.557
12.0	9.5	96,126.082	-62.013	97.589
14.0	11.5	116,363.151	-66.715	103.312
16.0	13.5	136,600.221	-70.087	106.556
18.0	15.5	156,837.291	-71.414	106.156
20.0	17.5	177,074.361	-71.290	103.859
22.0	19.5	197,311.431	-69.954	98.421
24.0	21.5	217,548.500	-67.671	92.330
26.0	23.5	237,785.570	-64.818	85.655
28.0	25.5	258,022.640	-61.087	78.567
30.0	27.5	278,259.709	-57.823	71.727
32.0	29.5	298,496.780	-54.073	65.094
34.0	31.5	318,733.849	-50.203	58.757
36.0	33.5	338,970.919	-46.340	52.862
38.0	35.5	359,207.989	-42.554	47.454
40.0	37.5	379,445.059	-38.879	42.440
42.0	39.5	399,682.129	-35.374	37.919
44.0	41.5	419,919.198	-31.993	33.790
45.5	43.0	435,097.400	-29.678	31.003

Temperature Only

Stresses in AB

$$M = \pm \frac{1,242,000}{43.0}(x - 2.5) \text{ lb ins}$$

x ft	M lb ins	σ top lb/in²	σ bottom lb/in²
2·75	± 7,220·930	± 6·130	± 9·878
4·0	± 43,325·581	± 36·208	± 58·205
6·0	± 101,093·023	± 80·683	± 124·385
8·0	± 158,860·465	± 119·130	± 190·319
10·0	± 216,627·907	± 150·333	± 238·517
12·0	± 274,395·349	± 177·017	± 278·572
14·0	± 332,162·791	± 190·440	± 294·408
16·0	± 389,930·233	± 200·067	± 304·167
18·0	± 447,697·674	± 203·854	± 303·025
20·0	± 505,465·116	± 203·449	± 294·756
22·0	± 563,232·558	± 199·686	± 280·946
24·0	± 621,000·000	± 193·170	± 263·554
26·0	± 678,767·442	± 185·024	± 244·504
28·0	± 736,534·884	± 174·375	± 224·273
30·0	± 794,302·326	± 165·657	± 204·748
32·0	± 852,069·767	± 154·353	± 185·814
34·0	± 909,837·209	± 143·305	± 167·723
36·0	± 967,604·651	± 132·280	± 150·847
38·0	± 1025,372·093	± 121·471	± 135·371
40·0	± 1,083,139·535	± 110·980	± 121·148
42·0	± 1,140,906·977	± 100·975	± 108·242
44·0	± 1,198,674·419	± 91·324	± 96·456
45·5	± 1,242,000,000	± 84·717	± 88·500

Shrinkage Only

Stresses in AB

$$M = -\frac{403,500}{430}(x - 2.5) \quad \text{lb-ins}$$

x ft	M lb-ins	σ top lb/sq in	σ bottom lb/sq in
2.75	−2,346.869	1.992	−3.210
4.0	−14,675.581	11.763	−18.910
6.0	−32,843.023	26.212	−42.035
8.0	−51,610.465	38.703	−61.831
10.0	−70,377.907	48.840	−77.489
12.0	−89,145.349	57.509	−90.502
14.0	−107,912.791	61.870	−95.810
16.0	−126,680.233	64.498	−98.818
18.0	−145,447.674	66.228	−98.447
20.0	−164,215.116	66.113	−95.760
22.0	−182,982.558	64.874	−91.273
24.0	−201,750.000	62.757	−85.625
26.0	−220,517.442	60.111	−79.434
28.0	−239,284.884	56.651	−72.862
30.0	−258,052.326	53.673	−66.518
32.0	−276,819.767	50.146	−60.367
34.0	−295,587.209	46.557	−54.490
36.0	−314,354.651	42.975	−49.043
38.0	−333,122.093	39.464	−43.979
40.0	−351,889.535	36.055	−39.358
42.0	−370,656.977	32.805	−35.166
44.0	−389,424.419	29.669	−31.336
45.0	−403,500.000	27.523	−28.752

Dead Load Moments ~ Curve L (Shore Spans)

CURVE L

X ft

Dist x ft from end of curve	wt of section in lbs/inch at x ft	3	5	7	9	11	13	15	17	19	21	23
2.75	300.000	4,72[illegible]	[illegible]	15,30[illegible]	22,[illegible]	[illegible]	[illegible]	431[illegible]	5,00[illegible]	5,850[illegible]	65,7[illegible]	72,900.000
4.00	1208.140		4,461[illegible]	48,3[illegible]	12,58[illegible]	101,231[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]598.560	274,771.920
6.00	1224.121			14,689.452	44,[illegible]	73,[illegible]	102,[illegible]	[illegible]	[illegible]3.472	190,[illegible]2.476	[illegible]41.780	249,720.[illegible]4
8.00	1250.013				15,002.[illegible]	43,[illegible]	75,[illegible]	[illegible]	[illegible]404	[illegible]	[illegible]002.028	225,002.34[illegible]
10.0	1283.269					15,423.22[illegible]	46,[illegible]	[illegible]	[illegible]	[illegible]	16[illegible],655.508	200,501.964
12.0	1320.742						15,[illegible]	[illegible]	[illegible]	[illegible]	14[illegible]45.536	174,344.544
14.0	1382.814							16,[illegible]	49,8[illegible]	[illegible]	[illegible]40.776	149,45[illegible]
16.0	1450.454								[illegible]	[illegible]	[illegible]7.240	121,838.13[illegible]
18.0	1532.9066									18,310.881	55,112.645	91,854.408
20.0	1625.1846										19,502.263	58,506.790
22.0	1736.4476											20,836.371
24.0	1866.1097											
26.0	2016.1013											
28.0	2200.5836											
30.0	2384.5264											
32.0	2614.3041											
34.0	2876.6532											
36.0	3158.2432											
38.0	3449.6982											
40.0	3872.2815											
42.0	4233.5501											
44.0	4718.3774											
45.25	1281.[illegible]											
TOTAL B.M.		900	22,562	73,[illegible]	153,885	264,803	40[illegible]	[illegible]655	790,820	1,034,7[illegible]	1,317,076	1,634,730

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Dead Load Moments ~ Curve L (Shore Spans)

25	27	29	31	33	35	37	39	41	43	45	45·5
80,100.000	87,300.000	94,500.000	101,700.000	108,900.000	116,100.000	123,300.000	130,500.000	137,700.000	144,900.000	152,100.000	153,900.000
303,695.240	332,618.640	361,572.400	390,465.360	419,388.720	448,332.080	477,285.440	506,158.800	534,[illegible]	564,[illegible]	592,928.800	609,159.720
279,099.[illegible]	308,478.492	337,[illegible].346	367,[illegible]	396,[illegible]	426,[illegible]	455,[illegible]	485,[illegible]	514,130.820	543,509.724	572,888.[illegible]	580,233.354
253,[illegible].652	285,[illegible].464	315,[illegible].276	345,[illegible]	375,[illegible]	405,[illegible].212	435,[illegible]	465,[illegible].476	495,[illegible]	525,[illegible]	555,045.172	567,[illegible]
231,348.420	262,144.876	293,641.352	323,[illegible]	354,[illegible]	385,[illegible]	416,427.[illegible]	447,273.[illegible]	478,[illegible]	509,[illegible]	534,812.980	547,584.574
206,043.55[illegible]	237,742.[illegible]	269,[illegible]	301,140.[illegible]	332,881.514	364,538.092	396,[illegible]	427,926.803	459,[illegible]	491,[illegible]	523,333.632	530,958.384
182,863.445	215,874.[illegible]	244,[illegible]	283,248.[illegible]	315,[illegible]	348,731.128	[illegible]	413,144.[illegible]	[illegible]	[illegible],560.272	514,770.[illegible]	523,081.[illegible]
156,644.03[illegible]	191,458.978	220,270.224	[illegible],081.[illegible]	319,692.416	[illegible],708.512	[illegible],514.408	[illegible]	435,[illegible]	[illegible]	[illegible].492	513,41[illegible]
128,596.171	165,337.934	202,[illegible].678	238,821.461	275,563.224	312,304.987	349,046.750	385,788.514	422,530.277	459,272.040	496,013.803	505,199.244
97,513.316	136,515.842	175,520.367	214,514.895	253,584.422	292,573.948	331,538.474	[illegible],543.[illegible]	[illegible],547.527	[illegible],552.054	[illegible],556.580	447,307.712
65,510.674	104,184.456	[illegible],558.238	187,532.021	224,205.803	[illegible],874.586	312,553.368	354,227.150	395,900.933	437,574.715	479,248.498	489,666.943
22,343.316	67,179.944	[illegible].582	156,758.415	201,634.[illegible]	246,516.480	291,113.118	335,[illegible]	380,636.377	[illegible],473.[illegible]	470,259.644	481,456.303
	24,193.288	72,574.863	120,466.438	[illegible],353.013	217,239.588	264,126.164	[illegible],512.739	362,849.314	411,235.889	459,672.464	471,769.108
		26,407.027	79,211.082	132,035.136	184,549.190	237,663.245	[illegible],477.299	[illegible],71[illegible].354	[illegible]	448,919.462	462,121.476
			25,614.317	86,022.950	143,[illegible].584	[illegible],720.219	[illegible]	[illegible],411.485	372,766.118	430,114.752	444,451.910
				31,345.673	[illegible].020	156,178.166	[illegible],769.712	[illegible].059	[illegible],352.405	408,143.752	423,[illegible]
					34,5[illegible]7.548	103,521.195	172,531.412	[illegible],553.189	310,568.306	374,583.[illegible]	396,831.382
						38,626.716	114,[illegible]	199,138.[illegible]	266,137.012	342,240.[illegible]	361,253.842
							41,936.380	125,489.175	209,981.942	293,474.649	314,972.838
								46,467.[illegible]	139,402.145	232,336.408	255,570.549
									51,462.661	154,387.[illegible]	180,114.104
										57,100.447	[illegible],656.130
											3,854.5[illegible]7
2,005,614	2,377,974	2,851,154	3,399,[illegible]	[illegible]77,526	4,621,654	5,338,314	[illegible]4,097	7,020,44 4	8,003,220	9,044,859	9,385,899

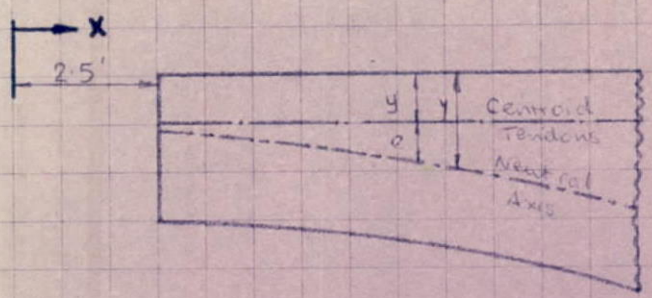
CURVE L (Span AB)

Dead Load B.M.

x	B.M calculated	B.M interpolated
2.75	—	400
3	900	—
4	—	11,731
5	22,562	—
6	—	47,968
7	73,374	—
8	—	113,630
9	153,885	—
10	—	209,344
11	264,803	—
12	—	335,902
13	407,001	—
14	—	493,828
15	580,655	—
16	—	685,488
17	790,320	—
18	—	912,541
19	1,034,761	—
20	—	1,175,919
21	1,317,076	—
22	—	1,478,403
23	1,639,730	—
24	—	1,822,672
25	2,005,614	—
26	—	2,191,779
27	2,377,944	—
28	—	2,629,549
29	2,881,154	—
30	—	3,140,230
31	3,399,306	—
32	—	3,688,417
33	3,977,528	—
34	—	4,299,591
35	4,621,654	—
36	—	4,979,984
37	5,338,314	—
38	—	5,736,656
39	6,134,997	—
40	—	6,577,571
41	7,020,144	—
42	—	7,511,682
43	8,003,220	—
44	—	8,549,040
45	9,094,859	—
45.5	9,385,899	9,385,899

Debonding Arrangements at Pin end of Units 1 and 5

Refer to diagram overleaf



56 No ½" ϕ strand.

An analysis willbe made of the following debonding arrangements :-

X = 2.5' – 13.5'

26 No. strands debonded. P = 576,000 lbf.
y = ~~6~~ 4.67"

X = 13.5' – 26.5'

20 No. strands debonded P = 691,200 lbf.
y = 5.40

X = 26.5' – 30.5'

14 No. strands debonded P = 806,400 lbf
y = 5.63"

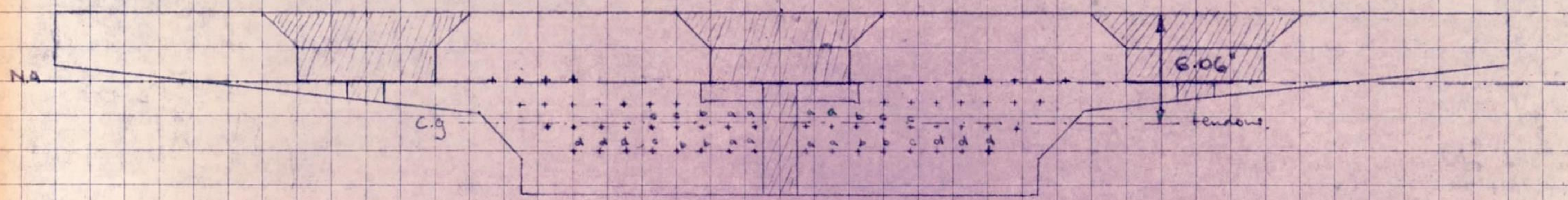
X = 30.5' – 32.5'

8 No strands debonded P = 921,600 lbf
y = 5.88"

X = 32.5' – Cantilever

P = 1,075,200 lbf
y = 6.06"

No strands debonded.



56 tendons ½" ϕ

8 @ ~~3.70"~~ 3.30 from topface

14 @ ~~5.20"~~ 4.8 " " "

18 @ ~~6.70"~~ 6.4 " " "

16 @ 8.25" " " "

$P_{initial}$ = 1,450,000 lbf.

P = 1,063,000 lbf

e = 10.57"

Force per strand = 19,200 lb

The following tendons to be debonded for these lengths from the ~~end of the beam~~ origin

						from face − 3'0"	C.g tendons topface
8	those marked	'a'	x	:	32.5'	29.5	5.88"
6	" "	'b'	x	:	30.5'	27.5	5.63"
6	" "	'c'	x	:	26.5'	23.5	5.40
6	" "	'd'	x	:	13.5'	10.5	4.67

END FRAME BEAM UNIT

Prestressing Stresses

x ft	Y ins	e = Y−y ins	Pe lb.ins	$\frac{Pe}{Z_{top}}$ lb/in²	$\frac{Pe}{Z_{bot}}$	$\frac{P}{A}$	$\frac{M_{D.L.}}{Z_{top}}$	$\frac{M_{D.L.}}{Z_{bot}}$
2.75	4.06	−0.61	−351,360	−300	+486	+1003		
4	4.10	−0.57	−328,320	−272	+442	+996		
6	4.19	−0.48	−276,480	−221	+354	+981		
8	4.32	−0.35	−201,600	−151	+241	+960		
10	4.50	−0.17	−97,920	−68	+108	+935		
12	4.68	+0.01	+5,760	+4	−6	+908		
14	4.99	−0.41	−283,392	−162	+252	+1040		
16	5.32	−0.08	−55,296	−28	+43	+993		
18	5.70	+0.30	+207,360	+94	−140	+941		
20	6.14	+0.74	511,488	+206	−298	+886		
22	6.64	+1.24	857,088	+304	−426	+829		
24	7.21	+1.81	1,251,072	+389	−531	+772		
26	7.83	+2.43	1,679,616	+458	−605	+714		
28	8.49	+2.86	2,306,304	+546	−702	+763		
30	9.25	+3.62	2,919,168	+607	−752	+703		
32	10.04	+4.16	3,833,856	+694	−836	+734		
34	10.88	+4.82	5,182,464	+816	−955	+770		
36	11.78	+5.72	6,150,144	+841	−959	+699		
38	12.72	+6.66	7,160,832	+848	−945	+633		
40	13.70	+7.64	8,214,528	+842	−919	+572		
42	14.73	+8.67	9,321,984	+825	−884	+516		
44	15.80	+9.74	10,472,448	+798	−843	+465		
45.5	16.63	+10.57	11,364,864	+775	−810	+431		

The stresses for the different loading cases are summarised in the two tables overleaf. Column 4 gives the stresses under the influence of Dead Load and Prestress only, and Columns 10 & 11 give the maximum and minimum stresses respectively for Dead Load and Live Load combinations. It will be seen that all stresses are safely within the range +0 → +2,500 p.s.i. Maximum Dead Load stresses have been checked for stress at transfer.

STRESS[illegible] FOR SHORE SPA[illegible] (Arch L) ~ Table A, σ_{top} p.s.i.

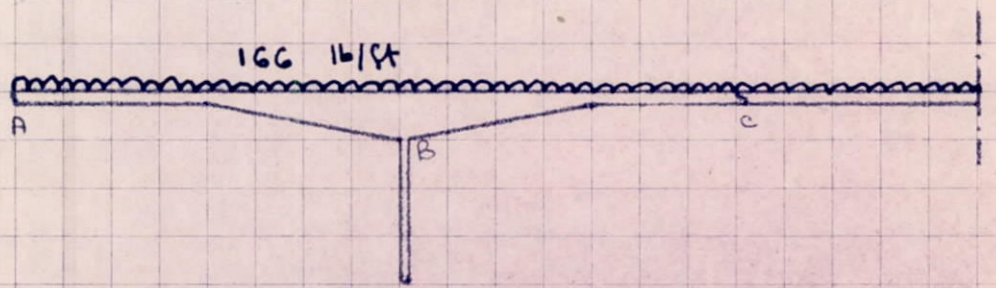
Col No.	1	2	3	4	5	6	7	8	9	10	11
X ft	Dead Load	$\frac{Pe}{Z_{top}}$	$\frac{P}{A}$	1 + 2 + 3	L.L. on AB	L.L. on BL	Temp. Contr.	Temp. Exp.	Shrinkage	4 + Σ +ve Stress	4 + Σ -ve Stress
2.75	- 0	- 300	+ 1003	+ 703	+ 26	- 2	+ 6	- 6	+ 2	+ 737	+ 695
4	- 10	- 272	+ 996	+ 714	+ 144	- 12	+ 36	- 36	+ 11	+ 905	+ 666
6	+ 38	- 221	+ 981	+ 722	+ 295	- 28	+ 80	- 80	+ 26	+ 1123	+ 614
8	- 85	- 151	+ 960	+ 724	+ 398	- 41	+ 119	- 119	+ 38	+ 1279	+ 564
10	- 145	- 68	+ 933	+ 720	+ 454	- 53	+ 150	- 150	+ 48	+ 1372	+ 517
12	- 217	+ 4	+ 908	+ 695	+ 478	- 62	+ 178	- 178	+ 57	+ 1408	+ 455
14	- 283	- 162	+ 1040	+ 595	+ 453	- 66	+ 190	- 190	+ 62	+ 1300	+ 339
16	- 351	- 28	+ 993	+ 614	+ 413	- 70	+ 200	- 200	+ 65	+ 1292	+ 344
18	- 416	+ 94	+ 941	+ 619	+ 356	- 71	+ 203	- 203	+ 66	+ 1244	+ 345
20	- 473	+ 206	+ 886	+ 619	+ 290	- 71	+ 203	- 203	+ 66	+ 1178	+ 345
22	- 524	+ 304	+ 829	+ 609	+ 221	- 69	+ 199	- 199	+ 64	+ 1093	+ 341
24	- 567	+ 389	+ 772	+ 594	+ 153	- 68	+ 193	- 193	+ 62	+ 1002	+ 333
26	- 598	+ 458	+ 714	+ 574	+ 87	- 65	+ 185	- 185	+ 60	+ 906	+ 324
28	- 622	+ 546	+ 763	+ 687	+ 27	- 61	+ 174	- 174	+ 57	+ 945	+ 452
30	- 652	+ 607	+ 703	+ 658	- 27	- 57	+ 165	- 165	+ 53	+ 876	+ 409
32	- 662	+ 694	+ 734	+ 766	- 74	- 54	+ 154	- 154	+ 50	+ 970	+ 484
34	- 677	+ 816	+ 770	+ 909	- 114	- 50	+ 143	- 143	+ 46	+ 1098	+ 602
36	- 681	+ 841	+ 699	+ 859	- 147	- 46	+ 132	- 132	+ 42	+ 1033	+ 534
38	- 680	+ 848	+ 633	+ 801	- 174	- 42	+ 121	- 121	+ 39	+ 961	+ 464
40	- 673	+ 842	+ 572	+ 741	- 194	- 38	+ 110	- 110	+ 36	+ 887	+ 399
42	- 664	+ 825	+ 516	+ 677	- 209	- 35	+ 100	- 100	+ 32	+ 809	+ 333
44	- 651	+ 798	+ 465	+ 612	- 218	- 32	+ 91	- 91	+ 29	+ 732	+ 271
45.5	- 640	+ 775	+ 431	+ 566	- 223	- 29	+ 85	- 85	+ 27	+ 678	+ 229

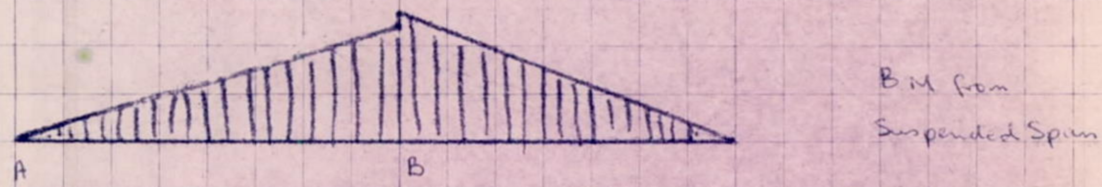
S.20

STRESSES FOR SHORE SPAN (Arch L) ~ Table B σ_{bot} p.s.i.

Col No	1	2	3	4	5	6	7	8	9	10	11
X ft	Dead Load	$\frac{Pe}{Z_{bot}}$	$\frac{P}{A}$	1 + 2 + 3	LL on AB	LL on BC	Temp Contn.	Temp. Exp.	Shrinkage	4 + Σ +ve Stress	4 + Σ −ve Stress
2.15	+ 0	+ 486	+ 1003	+ 1489	− 41	+ 3	− 10	+ 10	− 3	+ 1502	+ 1435
4	+ 16	+ 442	+ 996	+ 1454	− 220	+ 20	− 58	+ 58	− 18	+ 1532	+ 1158
6	+ 61	+ 354	+ 981	+ 1396	− 473	+ 45	− 129	+ 129	− 42	+ 1570	+ 752
8	+ 136	+ 241	+ 960	+ 1337	− 635	+ 66	− 190	+ 190	− 61	+ 1593	+ 451
10	+ 230	+ 108	+ 933	+ 1271	− 720	+ 83	− 238	+ 238	− 77	+ 1592	+ 236
12	+ 341	− 6	+ 908	+ 1243	− 752	+ 97	− 278	+ 278	− 90	+ 1618	+ 123
14	+ 438	+ 252	+ 1040	+ 1730	− 702	+ 103	− 294	+ 294	− 95	+ 2127	+ 634
16	+ 534	+ 43	+ 993	+ 1570	− 628	+ 106	− 304	+ 304	− 98	+ 1980	+ 540
18	+ 618	− 140	+ 941	+ 1419	− 529	+ 106	− 303	+ 303	− 97	+ 1828	+ 490
20	+ 685	− 298	+ 886	+ 1273	− 421	+ 103	− 294	+ 294	− 95	+ 1670	+ 463
22	+ 737	− 426	+ 829	+ 1140	− 312	+ 98	− 280	+ 280	− 91	+ 1518	+ 457
24	+ 774	− 531	+ 772	+ 1015	− 208	+ 92	− 263	+ 263	− 85	+ 1370	+ 459
26	+ 790	− 605	+ 714	+ 899	− 115	+ 85	− 244	+ 244	− 79	+ 1228	+ 461
28	+ 800	− 702	+ 763	+ 861	− 34	+ 78	− 224	+ 224	− 72	+ 1163	+ 531
30	+ 809	− 752	+ 703	+ 760	+ 33	+ 71	− 204	+ 204	− 66	+ 1068	+ 490
32	+ 804	− 836	+ 734	+ 702	+ 89	+ 65	− 185	+ 185	− 60	+ 1041	+ 457
34	+ 792	− 955	+ 770	+ 607	+ 133	+ 58	− 167	+ 167	− 54	+ 965	+ 386
36	+ 776	− 959	+ 699	+ 516	+ 168	+ 52	− 150	+ 150	− 49	+ 886	+ 317
38	+ 757	− 945	+ 633	+ 445	+ 194	+ 47	− 135	+ 135	− 43	+ 821	+ 267
40	+ 735	− 919	+ 572	+ 388	+ 212	+ 42	− 121	+ 121	+ 39	+ 763	+ 228
42	+ 712	− 884	+ 516	+ 344	+ 224	+ 37	− 108	+ 108	− 35	+ 713	+ 201
44	+ 687	− 843	+ 465	+ 309	+ 230	+ 33	− 96	+ 96	− 31	+ 668	+ 182
45.5	+ 668	− 810	+ 431	+ 289	+ 232	+ 31	− 88	+ 88	− 29	+ 640	+ 172

Bending Moments in Unit 1 due to Superload Only





These loading conditions have been evaluated for Live Loa[d] and Superload combined = 766 lb/ft. We can therefore take the proportion $\frac{166}{766}$ of the combined B.M. diagrams. The totals of the two diagrams above are tabulated overleaf.

Deflection of point A relative to Point B under Dead Load Only.

Using Mohrs Second Theorem of Area Moments, the procedure is similar to tha[t] for the suspended span and is tabulated overleaf.

Deflection, $\delta = \frac{\Sigma M}{E} = \frac{910,990}{5.75 \times 10^6} = 0.158''$ (downwards) or $\frac{5}{32}''$ do[wn]

TABLE G

SHORE SPAN AB SUPER ~~LIVE~~ LOAD MOMENTS & DEAD LOAD DEFLECTIONS

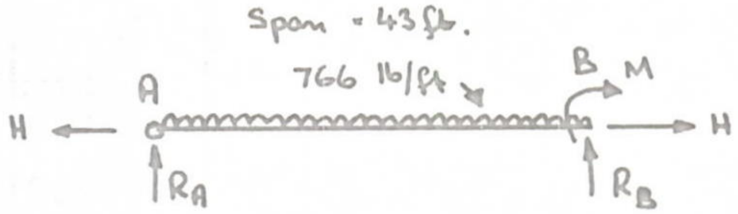
Col No	1	2	3	4	5	6	7	8	9	10	11	12	13
x ft	L.L. on AB lb in	L.L. on BC lb in	Sup Load on AB (21.67% ①+②)	D.L. Moment lb in	α in	P.e lb in	Total Moment ④+⑥ lb in	I in⁴	M/I = ⑦/⑧ lb/in³	Strip width in	Area of strip ⑨×⑩ lb/in²	Arm in	Moment ⑪×⑫ lb/in
2.75	-30,138	+ 2,529	-5980	+ 400		+351,360	74	4753	74.0	6	444.0	3	1,332.0
4	-172,213	+ 15,177	- 34,029	+ 11,731		+323,320		4,900	64.4	24	1722.6	18 ~~24~~	31,006
6	-369,657	+ 35,414	- 72,430	+ 47,968		+276,480		5,250	61.8	24	1483.2	42	62,294
8	-530,334	+ 55,651	-102,863	+ 113,630		+201,600		5,765	54.7	24	1312.8	66	86,645
10	-654,243	+ 75,889	- 125,329	+ 209,344		+97,920		6484	47.4	24	1137.6	90	102,384
12	-741,334	+ 96,126	- 139,827	+ 335,902		+ 5,760		7251	45.5	24	1092.0	114	124,488
14	-791,697	+ 116,363	- 146,344	+ 493,823		+ 283,342		8705	89.3	24	2143.2	138	295,762
16	-805,362	+ 136,600	- 144,920	+ 685,488		+ 55,296		10,362	71.5	24	1716.0	162	27,799
18	-782,199	+ 156,837	- 135,516 ~~- 118,180~~	+ 912,541		- 207,360		12,524	56.3	24	1351.2	186	251,323
20	- 722,267	+ 177,074	- 118,143	+ 1,175,919		- 511,458		15,257	43.5	24	1044.0	210	219,240
22	- 625,568	+ 197,311	- 92,803	+ 1,473,403		- 557,068		18,738	33.2	24	796.8	234	186,451
24	- 492,101	+ 217,548	- 59,495	+ 1,822,672		- 1,251,072		23,168	24.7	24	592.8	258	152,942
26	- 321,866	+ 237,785	- 18,220	+ 2,191,779		- 1,679,616		28,713	17.83	24	427.9	282	120,668
28	- 114,863	+ 258,022	+ 31,022	+ 2,629,549		- 2,306,304		35,843	9.0	24	216.0	306	6,609
30	+ 128,908	+ 278,259	+ 88,233	+ 3,140,230		- 2,914,168		44,504	5.0	24	120.0	330	39,600
32	+ 409,448	+ 298,496	+ 153,411	+ 3,688,417		- 3,833,856		55,407	- 2.6	24	+ 62.4	354	- 22,089
34	+ 726,755	+ 318,733	+ 226,557	+ 4,299,591		- 5,182,464		69,094	- 12.8	24	- 307.2	378	- 116,122
36	+ 1,080,830	+ 338,970	+ 307,670	+ 4,979,934		- 6,150,144		86,141	- 13.6	24	- 326.4	402	- 131,213
38	+ 1,471,673	+ 359,207	+ 396,751	+ 5,736,656		- 7,160,832		107,357	- 13.3	24	- 319.2	426	- 135,979
40	+ 1,899,284	+ 379,445	+ 493,800	+ 6,577,571		- 8,214,528		133,731	- 12.2	24	- 292.8	450	- 131,760
42	+ 2,363,664	+ 399,682	+ 598,730	+ 7,511,682		- 9,321,984		166,431	- 10.9	24	- 261.6	474	- 123,998
44	+ 2,864,811	+ 419,919	+ 711,801	+ 8,549,040		- 10,472,448		207,333	- 9.3	24	- 223.2	498	- 111,154
45.5	+ 3,264,800	+ 435,097	+ 801,767	+ 9,385,899		- 11,364,864		243,787	- 8.2	6	- 49.2	513	- 25,239

$\Sigma M = + 910,990$

Design of Abutment Pin

It is proposed to pin Units 1 & 5 to the abutment by stainless steel dowells passing horizontally into the abutment.

Consider AB as a separate span:-
(Continuous for Liveload only)



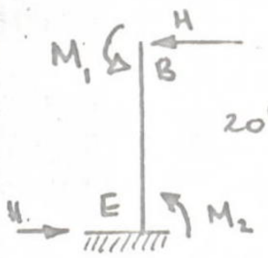
M =	3,264,800	(L.L. on AB)	
	435,000	(L.L. on BC)	
	1,242,000	(Temperature expansion)	
	4,941,800	lb.in.	

T.M.A. B :-

$$R_A \times 43 = +\frac{766 \times 43^2}{2} \mp \frac{4,941,800}{12}$$

$$R_A = \underline{6,900} \text{ lbf (say)}$$

From Equilibrium of Column :-



M_2 =	− 1,048,000	
	+ 1,950,000	
	+ 398,600	
	1,300,600	lb.in

T.M.A. E.

$$12 \times 20 \times H = 4,941,800 - 1,300,600$$

$$H = \underline{15,171} \text{ lbf.}$$

Dowells are Spring temper EN 56D. Tensile Strength 95 T/in²
~~Shear Strength [illegible] T/in²~~ ([illegible])

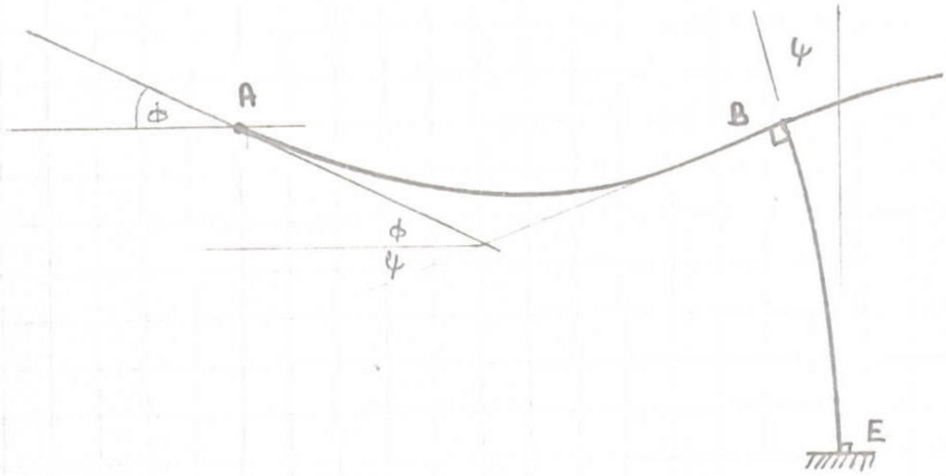
8 No 3/8" ϕ dowells = 0.884 in²

∴ Tensile stress on dowells $= \frac{15,171}{0.884} = 17,200$ p.s.i.

Shear stress $= \frac{6,900}{0.884}$ 7,800 p.s.i.

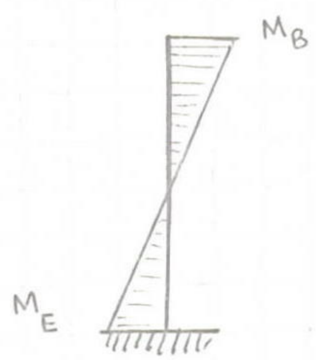
} O.K.

Maximum rotation Expected at Abutment Pin



Bending Moment on Column

M_E = 1,048,000
398,600
129,350
1,575,950 lb.in.



M_B = 3,264,800 (L.L. on AB)
1,242,000 (Temp. contr.)
403,500 (shrinkage)
+ 5,109,800 lb in

By Mohr I , ψ = Area $\frac{M}{EI}$ diag from E to B

Column I taken from p. & $E_c = 3 \times 10^{+6}$ lb/in² $\psi = \frac{736}{3\times10^6} = 0.000245$ rads.

ft from base	M lb in	I in⁴	$\frac{M}{I}$ lb/in³	Length strip L in	$\frac{M.L}{I}$
0	−1,575,950	76,182	−20.68	12	−24.816
2	−907,375	101,769	−8.92	24	−214.08
4	−238,800	132,519	−1.80	24	−43.2
6	+429,775	168,740	2.55	24	61.2
8	+1,09,8350	211,100	5.20	24	124.8
10	+1766,925	256,780	6.88	24	165.12
12	+2,435,500 ~~+2,366~~	312,327	7.80	24	187.2
14	+3,104,075	375,162	8.27	24	198.48
16	+3,772,650	446,691	8.45	24	202.80
18	+4,441,225	526,637	8.43	24	202.32
20	+5,109,800	615,883	8.30	12	99.60

$\Sigma \frac{M_L}{I} = 736.08$

Bending Moment on AB

Combine L.L. on AB
Temp. contraction
Shrinkage

x ft	M L.L. on AB lb in	M Temp contr. lb in	M Shrinkage lb in	ΣM	$\frac{\Sigma M}{I}$	Strip L ins	$\Sigma \frac{M.L}{I}$
4	−172,000	− 43,000	−14,000	−229,000	− 46.73	30	− 1,401.90
6	−370,000	− 101,000	− 38,000	−503,000	− 95.81	24	− 2,299.44
8	−530,000	− 159,000	− 52,000	−741,000	− 128.53	24	− 3,084.72
10	−654,000	− 217,000	− 70,000	−941,000	− 145.10	24	− 3,482.40
12	−741,000	− 274,000	− 89,000	− 1,104,000	− 152.23	24	− 3,653.52
14	−791,000	− 332,000	− 108,000	− 1,231,000	− 141.41	24	− 3,393.84
16	−805,000	− 390,000	−127,000	− 1,322,000	− 127.58	24	− 3,061.92
18	− 703,000	− 478,000	− 145,000	− 1,296,000	− 103.48	24	− 2,483.52
20	−722,000	−506,000	− 164,000	− 1,392,000	− 91.24	24	− 2,189.76
22	−625,000	− 563,000	− 183,000	− 1,371,000	− 73.17	24	− 1,756.08
24	− 492,000	− 621,000	− 202,000	− 1,315,000	− 56.76	24	− 1,362.24
26	−321,000	− 679,000	− 221,000	− 1,220,000	− 42.49	24	− 1,019.76
28	− 114,000	− 737,000	− 239,000	− 1,090,000	− 30.41	24	− 729.84
[illegible]	+ 129,000	− 794,000	− 258,000	− 923,000	− 20.74	24	− 497.76
[illegible]	+ 409,000	− 852,000	− 276,000	− 719,000	− 12.98	24	− 311.52
34	+ 726,000	− 910,000	− 296,000	− 480,000	− 6.95	24	− 166.80
36	1,081,000	− 968,000	− 315,000	− 202,000	− 2.34	24	− 56.16
38	1,472,000	− 1,025,000	− 333,000	+ 114,000	+ 1.06	24	+ 25.44
40	1,899,000	− 1,083,000	− 352,000	+ 464,000	+ 3.47	24	+ 83.28
42	2,363,000	− 1,141,000	− 371,000	+ 851,000	+ 5.11	24	+ 122.64
44	2,865,000	− 1,200,000	− 390,000	+ 1,275,000	+ 5.54	30	+ 166.20

$$\Sigma\left(\Sigma \frac{ML}{I}\right) = -30{,}553.62$$

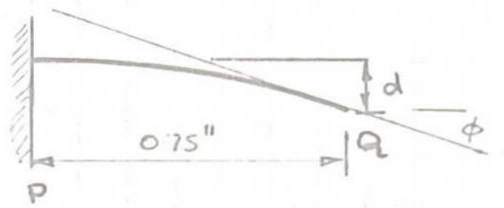
Change in slope from A-B, $\phi + \psi = \frac{30,553.62}{5.75 \times 10^6} = \underline{\underline{0.00531 \text{ rads}}}$

Then $\phi = 0.00531 - 0.00025$
$= \underline{\underline{0.00506 \text{ rads}}}$

Stresses on Dowel.

(a) due to rotation

3/8" ϕ bars.
$\phi = 0.00506$ rads



Approximately, $d = \frac{1}{2} . 0.75 . 0.00506$

$= 0.0019$ ins

Bending Moment at P $= \frac{3EI.d}{L^2} = \frac{3 \times 28 \times 10^6 \times \pi \times (3/8)^4 \times 0.0019}{(0.75)^2 \times 64}$

$= 275.4$ lb.in.

Stress/bar $= \pm \frac{275.4 \times 64 \times (3/8)}{\pi \times (3/8)^4 \times 2 \times 8} = \pm$ 6,650 p.s.i./bar.

(b) Tensile stress (from before) = −17,200 p.s.i

Shear Stress (from before) = 7,800 p.s.i.

(a) & (b) are additive = f_t = −23,850 p.s.i. & −10,550 p.s.i.

Principal Stresses (negative stress is criterion)

$\sigma_p = \frac{-23{,}850}{2} \pm \left(\sqrt{(-23{,}850)^2 + 4.(7{,}800)^2}\right) 1/2$

$= -11{,}925 \pm 14{,}250$

$\sigma_p = +2{,}325$ and $-26{,}175$ p.s.i.

i.e. 11.7 tons/in²

Selection of Suitable Stainless Steel for Dowel.

Requisite properties

Minimum proof stress		25 T/in²
Minimum shear stress		8 T/in²
Fatigue limit	>	12 T/in²
Resistance to atmospheric corrosion		Absolute.

Most stainless steels are considerably stronger than our requirements, but most of them are also liable to cold work-harden and brittle fracture. We require a steel that combines good fatigue strength (10^6 reversals of stress per year may be expected) with no susceptibility to fatigue corrosion. A good general steel for this purpose is EN56D - Spring temper. However, in particular, a steel as developed by Firth-Vickers, FV520B is probably the most suitable. This steel would be used in the '620°C overaged' condition, combining maximum ductility and fatigue strength with corrosion resistance.

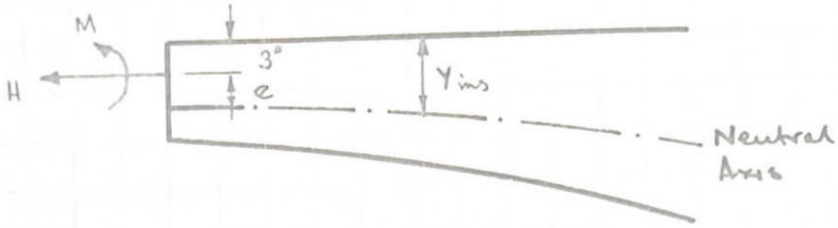
Mechanical Properties

		EN56D (Spring temper)	FV520B (620°C overaged)
0.2% proof stress,	T/in²		35
U.T.S.	T/in²	95	55-65
Youngs Modulus	T/in²		
Fatigue Limit	T/in²		38
Hardness	H.V.	480	270 - 320
Corrosion Resistance in 0.25% H_2SO_4	(loss in gm/cm²/24hrs)	0.010	0.0000

Secondary Stresses in Span AB.

Consider the loading conditions used in the design of the abutment pin:

The beam is subjected to secondary stresses created by the horizontal force at the pin. H ; and a moment M, caused by the resistance of the hinge.



M = 275 lb.in. – creates insignificant stresses and will be ignored

H = 15,200 lb.

Comparison of the stresses evaluated in cols ⑤ & ⑥ with those in Tables A & B cols ⑩ & ⑪ shows no excessive combined stresses occur.

	①	②	③	④	⑤	⑥
X ft	e ins	He/Zt psi	He/Zb psi	H/A psi	② + ④ = σ_{top} psi	③ + ④ = σ_{bot} psi
2.5	1.0	−13	+21	−26	−39	−5
4.0	~~1.0~~ 1.1	−14	+22	−26	−40	−4
6.0	~~1.1~~ 1.2	−15	+23	−26	−41	−3
8.0	~~1.2~~ 1.3	−15	+24	−25	−40	−1
10.0	~~1.3~~ 1.5	−16	+25	−25	−41	0
12.0	~~1.5~~ 1.7	−17	+26	−24	−41	+2
14.0	~~1.7~~ 2.0	−17	+27	−23	−40	+4
16.0	~~2.0~~ 2.3	−18	+27	−22	−40	+5
18.0	~~2.3~~ 2.7	−19	+28	−21	−40	+7
20.0	~~2.7~~ 3.1	−19	+27	−19	−38	+8
22.0	~~3.1~~ 3.6	−19	+27	−18	−37	+9
24.0	~~3.6~~ 4.2	−20	+27	−17	−37	+10
26.0	~~4.2~~ 4.8	−20	+26	−16	−36	+10
28.0	~~4.8~~ 5.5	−20	+25	−15	−35	+10
30.0	~~5.5~~ 6.2	−20	+24	−13	−33	+11
32.0	~~6.2~~ 7.0	−19	+23	−12	−31	+11
34.0	~~7.0~~ 7.9	−19	+22	−11	−30	+11
36.0	~~7.9~~ 8.8	−18	+21	−10	−28	+11
38.0	~~8.8~~ 9.7	−17	+19	−9	−26	+10
40.0	~~9.7~~ 10.7	−17	+18	−8	−25	+10
42.0	~~10.7~~ 11.7	−16	+17	−7	−23	+10
44.0	~~11.7~~ 12.8	−15	+16	−7	−22	+9
45.5	13.6	−14	+15	−6	−20	+9