UNIVERSITY OF LONDON

GENERAL CERTIFICATE OF EDUCATION EXAMINATION

Advanced Level

SUMMER, 1960

APPLIED MATHEMATICS.—I

WEDNESDAY, June 22.—Afternoon, 2 to 5

Not more than EIGHT *questions are to be attempted.*

1. (i) A uniform lamina of weight W has the form of a trapezium in which the lengths of the parallel sides are in the ratio 2 : 3. Show that the centre of gravity of the lamina coincides with that of four particles of weights $\frac{W}{3}$, $\frac{2W}{15}$, $\frac{W}{3}$ and $\frac{W}{5}$ placed at the corners of the lamina.

(ii) A uniform equilateral triangular lamina ABC of weight $2W$ is suspended from A and is at rest with weights W and $3W$ attached at B and C respectively. Find the inclination of BC to the horizontal.

2. Two small rings of weights $3w$ and $5w$ are capable of sliding on a smooth circular wire of radius a fixed in a vertical plane. The rings are connected together by a light inextensible string of length $\frac{8a}{3}$ which passes over a smooth peg fixed at a height $\frac{a}{3}$ vertically above the highest point of the wire. The rings rest on opposite sides of the vertical through the peg. Find the reaction of the wire on each ring and show that the tension in the string is $\frac{15w}{4}$.

If the wire is uniform and of weight w, find the horizontal and vertical components of the external force required to keep the wire in position, indicating the directions clearly.

3. A framework is formed of five freely-jointed coplanar light rods PQ, QR, RS, SP and QS, with P and R on opposite sides of QS, the angles PQS, RQS, PSQ and RSQ being 60°, 75°, 90° and 30° respectively. The framework is at rest suspended from Q with loads 1 lb, 1 lb and w lb suspended from P, S and R respectively. Show that, if QS is vertical, $w = 2\sqrt{3}$ lb.

Draw the force diagram for this case. Show that the tension in the rod QR is $(\sqrt{3} - 1)\sqrt{6}$ lb and find the magnitude and nature of the stresses in the remaining rods.

4. A uniform beam AB of weight W rests horizontally with A attached to a light rope which passes over a smooth fixed pulley and supports a heavy particle hanging freely, whilst B is in contact with a rough plane of inclination α to the horizontal. The rope, beam and the line of greatest slope through B are in the same vertical plane. If λ is the angle of friction and B is about to slip downwards, find the weight of the particle and draw figures for the cases (*a*) $\lambda < \alpha$, (*b*) $\lambda > \alpha$.

In case (*a*) show that, if $\alpha > \frac{\pi}{2} - \lambda$, B cannot slip upwards, whatever the weight of the particle.

5. A bird flies in a straight line with uniform velocity u in an upward direction making an angle β with the horizontal. At the instant when the bird is at a height h vertically above a boy on the ground the boy throws a stone at an angle of elevation α. Show that, whatever the velocity of projection, the stone cannot hit the bird unless

$$\tan \alpha \geqslant \frac{\sqrt{2gh}}{u} \sec \beta + \tan \beta .$$

If the stone merely grazes the bird so that the motion of neither is appreciably disturbed, show that, in general, the bird will be hit again.

6. A light spring stands upright with its lower end fixed and its upper end attached to a light horizontal platform. A particle of mass m is placed on the platform and, when at rest, the spring is found to be compressed a distance a. The platform and particle are depressed a further distance $(c - a)$ and the system is released from rest. Show that, in the subsequent motion, when the total compression is a distance x, the downward acceleration of the particle is $g\left(1 - \frac{x}{a}\right)$.

Find the maximum speed of the particle and show that, if $c > 2a$, the particle will leave the platform.

7. A smooth straight heavy tube of length $2a$, with both ends closed, lies on a horizontal table and contains a particle of equal mass at its mid-point. The particle is projected along the tube with velocity u and e is the coefficient of restitution between the particle and either end of the tube. The velocities of the particle after the first and second impacts are u_1 and u_2 whilst the corresponding velocities of the tube are v_1 and v_2, all velocities being measured in the direction of u. Find u_1, u_2, v_1 and v_2.

Show that the particle passes the mid-point of the tube, moving in its original direction relative to the tube, after a time $\frac{a}{u}\left(1 + \frac{1}{e}\right)^2$ and find how far the tube has moved in this time.

8. Find, from first principles, the moment of inertia of a uniform rod of mass m and length l about an axis through one end perpendicular to the rod.

If the rod is fixed at one end and oscillates freely in a vertical plane through a small angle, show that the period of oscillation is $2\pi\left(\frac{2l}{3g}\right)^{\frac{1}{2}}$.

Two uniform rods, of lengths $8\frac{1}{3}$ ft and 3 ft, hang vertically, a small distance apart, with their upper ends fixed and their lower ends at the same horizontal level. Keeping the rods in the same vertical plane the lower ends are slowly brought together and then released. Find the time that elapses from release until these ends meet again for the first time.

[Take g as 32 ft/sec.^{2}]

9. State the conditions of equilibrium of a body floating freely in a liquid.

A uniform right prism has a triangular cross-section ABC in which $CA = 4$ in., $CB = 5$ in. and the angle $C = 90°$. The prism floats in water with its triangular faces vertical, C immersed, and with the water surface bisecting CA and intersecting CB. Show that there are two positions of equilibrium and find the specific gravity of the prism in each case.

Find the thrust on either triangular face when the specific gravity has its larger value.

10. Write down formulae for the mean M and standard deviation σ of a set of numbers which occur with given frequencies.

If s is the root mean square deviation measured from an assumed mean A, show that

$$\sigma^2 = s^2 - (M - A)^2 .$$

Taking 17 as an assumed mean, find the true mean and the standard deviation in the following case:—

Number	15	16	17	18	19	20
Frequency	4	10	13	8	3	2